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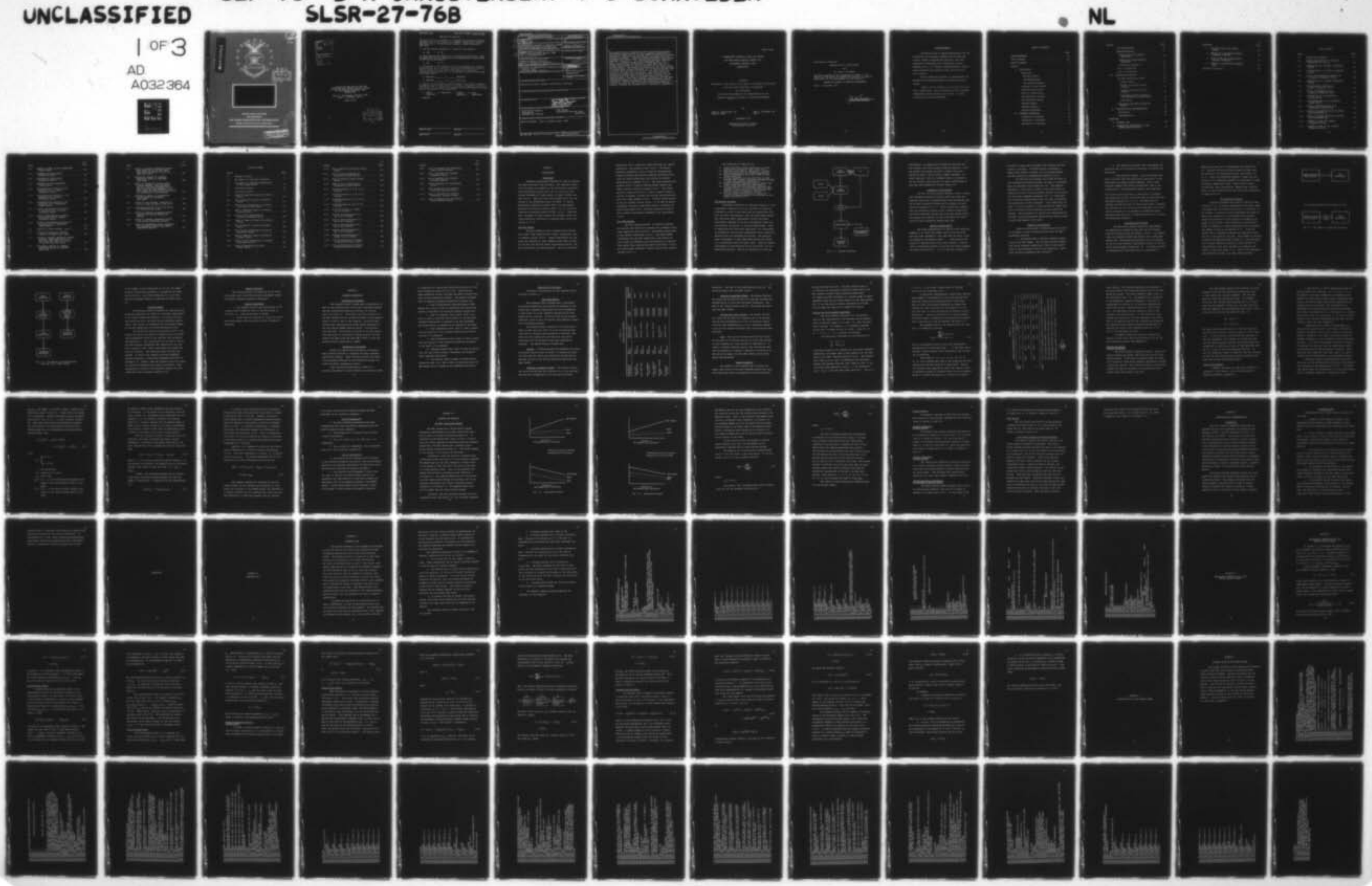

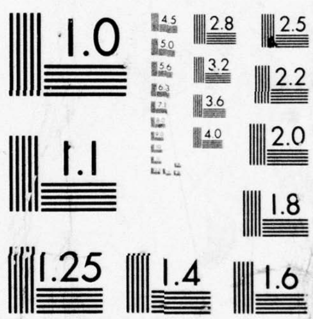
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A COMPARATIVE ANALYSIS OF THE DO41 SYSTEM AND TIME SERIES ANALYSIS MODELS FOR FORECASTING REPARABLE ITEM GENERATIONS

Bruce R. Christensen, Captain, USAF
Gene J. Schroeder, GS-12

SLSR 27-76B

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This research effort compared the D041 Single Moving Average forecasting method used to forecast reparable generations of recoverable items with the Box and Jenkin's Time Series Analysis forecasting methods. Five artificially generated stochastic processes were used to model the possible reparable generations observed in practice: (1) a Poisson process with a constant mean, (2) a Poisson process with a decreasing mean, (4) a Poisson process with an alternating linear mean, and (5) a process whose values are the sine function of the output of a Poisson process. The research concluded that the D041 forecasting method made unbiased forecasts for the Poisson process with a constant mean and the sine function, but made biased forecasts for the other three processes. Time Series Analysis forecasting methods were only used to make forecasts for the processes that were found to be biased using the D041 forecasting method. Time Series Analysis forecasting methods made unbiased forecasts for the processes whose means were linearly increasing, linearly decreasing, and alternating linearly. A guide for using the Box and Jenkin's Time Series Analysis computer programs was developed and is contained in Appendix E.

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A COMPARATIVE ANALYSIS OF THE D041 SYSTEM AND TIME SERIES ANALYSIS MODELS FOR FORECASTING REPARABLE ITEM GENERATIONS

A Thesis

Presented to the Faculty of the School of Systems and Logistics

of the Air Force Institute of Technology

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In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Logistics Management

By

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This thesis, written by

Captain Bruce R. Christensen

and

Mr. Gene J. Schroeder

has been accepted by the undersigned on behalf of the faculty of the School of Systems and Logistics in partial fulfillment of the requirements for the degree of

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CHAPTER I

INTRODUCTION

Background

Reparable generations forecasts are used in managing the USAF recoverable item inventory. The reparable generations forecasts are computed by the Recoverable Consumption Item Requirements System (D041) and are interfaced with various Management of Items Subject to Repair (MISTR) systems (9:1-1). There are three data systems which form the nucleus of the mechanized portion of MISTR: (1) Repair Requirements Computation System (D073), (2) Requirements Scheduling and Analysis System (G019C), and (3) Contracts Schedules and Analysis System (G019F) (8:2-1). These systems interface in such a manner that forecast errors can cause substantial difficulty in planning depot level maintenance (2; 3; 4).

The D041 System

The D041 system is used to identify from 125,000 line items, those requiring buy, repair, termination, and/or disposal actions (9:1-1). Repair requirements are computed and projected by stock numbers within family groups and are fed into the D073 repair requirements system. The D041 computations are made quarterly. The first quarterly

computation (FY-1) identifies items requiring buy, repair, termination, and disposal action (9:1-1). The second quarterly computation (FY-2) is used for adjusting previously initiated item actions, updating workload projections for annual negotiation, buy actions in process, termination and disposal (9:1-1). The third quarterly computation (FY-3) is used to develop budget requests and update logistic actions in process (9:1-1). The fourth quarterly computation (FY-4) is used to update logistics actions in process, initiate new actions, and provide data input into other systems (9:1-1). The D041 system applies to all Air Logistic Centers (ALCs). It is used to manage expendable investment recoverable spares; to develop and adjust requirements data, including maintenance factors; and to compute recoverable consumption item requirements (9:1-1).

The MISTR Systems

The D073 system provides: "a quarterly forecast for planning and negotiation purposes and a biweekly short-range computation which acts as a workloading system based on the quantity negotiated [8:2-1]." The G019C system "interfaces with the D073 system and is the vehicle by which items are scheduled into and production is reported from organic SRAs (Specialized Repair Activities) [7:2-1]." The G019F system "performs a similar function for contract sources [8:2-1]."

The objectives of MISTR are to:

(1) Project quarterly repair quantities on a long-range basis (current and next fiscal year).
(2) Negotiate with organic, contractual, or inter-service agencies for all funded requirements.
(3) Workload maintenance facilities based on negotiated quantities.
(4) Provide necessary tools to facilitate optimum internal depot repair scheduling through portrayal of asset and component availability.
(5) Provide automatic requisitioning of current and longer range component part requirements.
(6) Provide operating and management personnel the necessary tools to monitor scheduling and repair progress through preparation and analysis of a complete range of management products.
(7) Provide optimum communication channels and operating frequency between the customer and the repair activity [8:2-1].

The Systems Interface

According to AFLCR 65-12 (Reference Figure 1.1) the requirements computed by D041 for inclusion in MISTR are projected by actual stock number and are mechanically fed into the prime Item Manager's (IM) D073 requirement system. The long-range repair requirements include the annual requirements computed by the D041 requirements system and are provided, by quarter, to the D073 MISTR system. Prior to negotiating the long-range requirement, the requirement is reviewed for accuracy and completeness and to insure all technical data are available and current. The quarterly repair requirement is then negotiated between the IM/ALC, and a preassigned source of repair (SOR). These are yearly requirements at the beginning of the fiscal year, but are negotiated quarterly. The act of negotiation aligns the

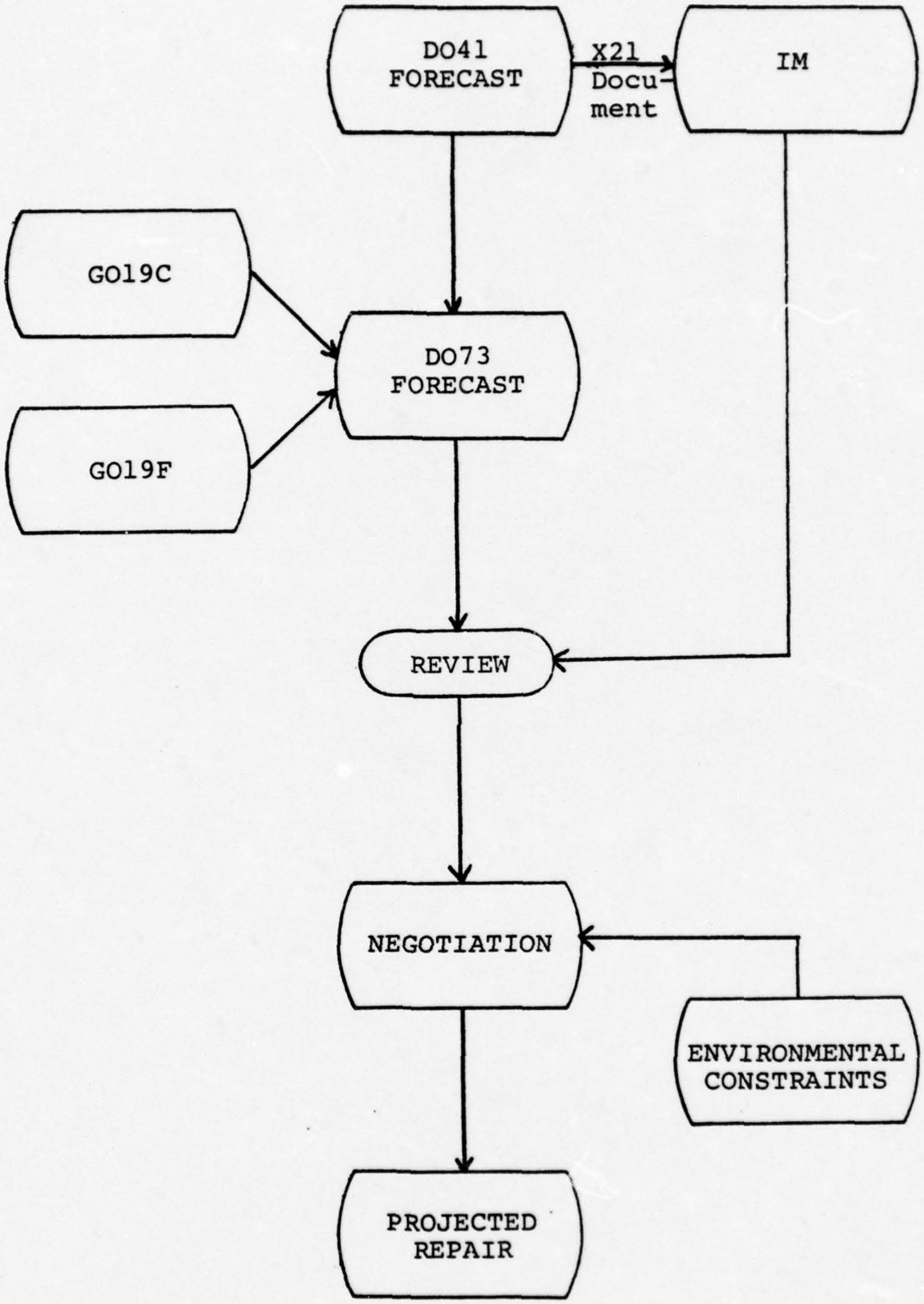


Fig. 1.1. Systems Interface

requirements, as computed and reflected by the D041 and D073 systems, with the available funds and manhours. Various systems and methods are used to predict the availability of assets, parts, skills, support equipment, and other resources. The X-21 document, which is provided to the IM, reflects eight quarters of repair requirements as they have been computed by the D041 system (5).

Statement of the Problem

The D041 and D073 systems interface in such a manner that reparable generation forecast errors can cause substantial difficulty in planning depot level maintenance and the current forecasting methods may not present the most accurate forecasts (2; 3; 4). Such a possibility suggests that the differences between actual reparable generations and those forecasted through use of the D041 method need to be analyzed and compared with the differences that would exist if other forecasting methods were used.

Research Justification

The prime responsibility of the Air Force Logistics Command is to insure that combat units of the Air Force have the right equipment at the right time and place (8:22). To accomplish this mission, AFLC ". . . must maintain the ability to replenish base stock levels through a constant flow of recoverable material to and from our global deployed Air Force units [8:2-2]." It is in the best interest of

the USAF to manage the recoverable item inventory as efficiently as possible. The MISTR system is designed to manage items subject to repair, and its success depends upon the accuracy of preproduction planning (8:2-2).

The D041 and D073 systems, which form the heart of the preproduction planning process, are used in the management of recoverable item inventories (1). The question addressed in this thesis is "What is the significance to production planning of the difference between the forecasted and the actual reparables generated (3)?" An understanding of the D041 computational model will help in analyzing the significance of that difference. After describing the model, it will be possible to make statistical comparisons with the forecast errors generated by other predictive methods. This knowledge will enhance the understanding of the D041 inventory management system and may indicate areas in which improvements can be made.

Areas of Investigation

There are three areas of investigation that relate to the impact of D041 forecasting errors upon the depot maintenance function:

1. The method used to forecast reparable generations by the D041 system. How do these forecasts compare with actual reparable generations? What is the significance of the differences found in the comparison? Would other predictive methods be more accurate?

2. The negotiation process. What requirements are sacrificed due to the forecast limitations of manhours and facilities?

3. The production agency. How are available manhours, facility time, and spare parts availability in the maintenance and supply systems computed? How do these forecasts compare with actual availability? What is the significance of the differences found in the comparison?

The volume of data available in these areas is enormous and comparisons of data in the past have not been made with meaningful analysis (2; 3; 4; 5). The focus of this research was upon the first of these three areas. Although the research did not focus upon the negotiation and the production processes, it is necessary to understand these processes since they are an integrated portion of the inventory management system(3).

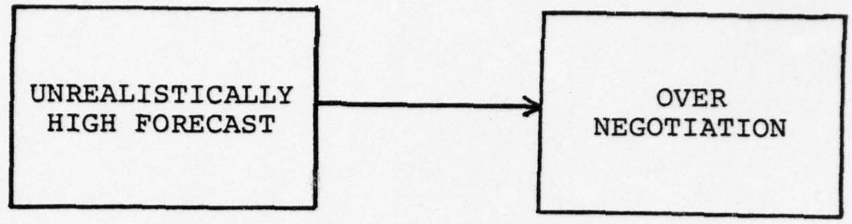
The Negotiation Process

The negotiation process aligns the requirements, as computed and reflected by the D041 and D073 systems, with the available funds and manhours (8:2-3). The negotiations are conducted between the IM/ALC, and a preassigned organic SOR. Negotiated quantities are to be a statement of the IM's most essential requirements limited by the most recent dollar/manhour constraints (5). A screening process is accomplished by both sides prior to negotiation to

determine the best mix of requirements and capabilities (5). Forecast errors by the D041 system can impact upon the accuracy of the negotiation effort (see Figure 1.2). (3; 5). Since the negotiation process is based upon projections, it is sometimes necessary to renegotiate in order to maintain supportable items. Factors such as asset availability, parts availability, demand rates, modifications, or other production-limiting factors may necessitate a renegotiation (5).

The Production Process

Inaccurate forecasts will lead to inaccurate negotiations (3; 5). Inaccurate negotiations will have a negative impact upon the production function (5) (see Figure 1.3). For example, a negotiation which establishes the number of units to be repaired at a level higher than necessary causes the forecasts of parts required to accomplish the repair to be too high. The Directorate of Distribution (D/DS) is required to allocate these parts for the negotiated number of units to be repaired (3; 5). Since the organic SOR is incapable of meeting the negotiated level, not all of the parts will be used. This results in the misallocation of operating funds by D/DS (5). This misallocation of funds reduces the amount of funds available for requisitioning other parts necessary to accomplish the repair of other line items (3; 5). This in turn decreases the production of other line items requiring repair.



If IM and SOR Recognize the Forecast as High

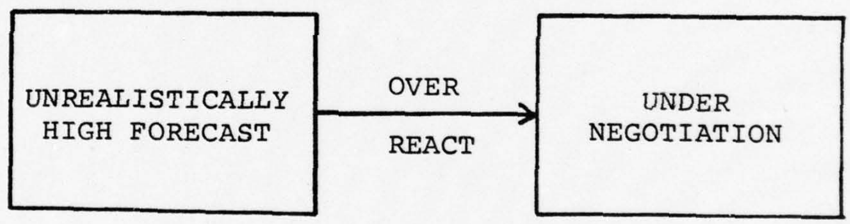


Fig 1.2. The Impact of Inaccurate Forecasts

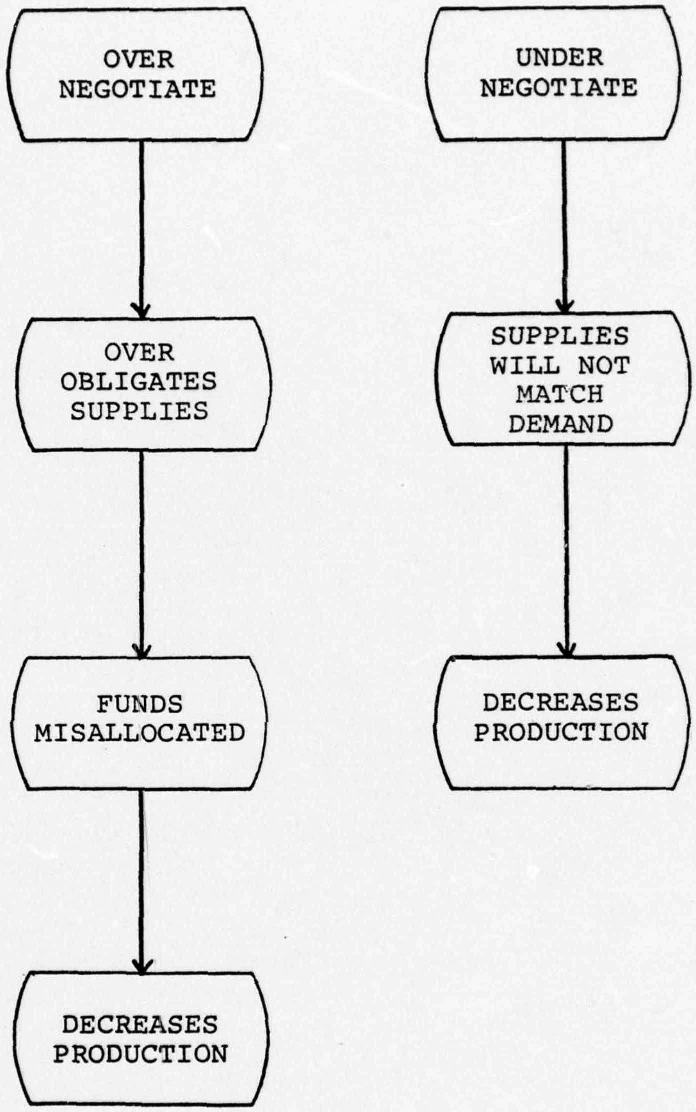


Fig. 1.3. The Impact of Inaccurate Negotiations of the Supply System

If the number of units negotiated is too low, the number of parts forecasted as necessary to accomplish the repair will be too low. The actual production of a line item would be decreased because of a shortage of repair parts (5).

Forecast Method

The preceding section discussed the negotiation and production processes, the impact of inaccurate negotiations on the supply system and production, and identified forecast errors as one of the major factors affecting negotiations. This section is concerned with the forecast method of the D041 system and its related forecasting errors.

The forecasting method and procedures found in the current directives are typically translations of the underlying mathematical model of the technique into procedural instructions (7). The instructions are provided to the IM and other Materiel Management (MM) agencies for use; and, over a period of time, these instructions have become the focus of attention. The underlying model has been suppressed. As long as the technique provides reasonably accurate forecasts, no particular management attention is focused on it. However, when the forecast errors are large, all levels of management must answer the question, "What significance is there in the difference between the forecasted and actual reparables generated (3)?"

Research Question

What characteristics are exhibited by the underlying model used in the D041 inventory management system to generate reparable generation forecasts?

Research Hypotheses

1. The expected value of the distribution of the forecast errors presently found in the D041 system is not zero, i.e., that the forecast is biased.

2. Other predictive methods for inventory management might exhibit less bias than the D041 forecasting technique.

CHAPTER II

RESEARCH METHODOLOGY

Population of Interest

This research effort focused upon an examination of some of the characteristics of the D041 forecasting method. The D041 system makes quarterly forecasts for 125,000 master line items (2). Recoverable item data currently in the D041 system could not be made available in the required format in time to use in this research. Artificial data patterns were generated using random number generators of the Time Sharing CREATE Computer System. Five patterns were generated. Each pattern consisted of 120 data-points (the equivalent of 120 months of demand history) and each pattern was begun using the same SEED (12345) so that the process could be replicated if needed.

Definition of Variables

Within the complex D041 forecasting environment, many variables interact to influence the depot reparable generations forecast. These variables include the subjective variables controlled by the maintenance manager and random variables introduced by nature.

Under procedures established by AFLCM 57-3, periodic adjustments of these subjective variables are made

to compensate for unpredicted variations and errors in the D041 forecasting system (9:5-27,5-31). Operationally, these subjective factors influence the depot workload forecast, and the negotiation process. The primary variables used in computing reparable generations forecasts are:

1. Total Organizational Field Maintenance (OFM) Demand Rate, which is used in conjunction with the Future Program to determine the projected reparable generations.

2. Base Not Reparable This Station (NRTS) rate, which is used to determine which portion of the projected reparable generations is expected to be base processed, and which portion is expected to be repaired at the depot.

3. Base Condemnation Rate, which is used to determine the fraction of the reparable generations processed at base to be condemned.

4. Depot Condemnation Rate, which is used to determine the fraction of the reparable generations processed at the Depot to be condemned.

5. OFM Depot Demand Rate, which is used on conjunction with the Future Program to determine the projected depot reparable generations.

6. The Program, which is based on flying hours, historical data, etc., and is used in conjunction with the OFM demand rates to determine the reparable generations.

Tabulation of Variables

Variables of significance in this research effort are shown in Table 2.1.

Data Requirements

This research effort focused upon a time-series forecasting methodolcgy and required 36 forecasts of projected depot reparable generations and the corresponding 36 data points of actual reparable generations for each demand pattern analyzed. Thirty-six data points yielded sufficient terms so that the Central Limit Theorem's assertion of normality applied.

Each demand pattern consisted of 120 data-points which allowed using the first 24 as the forecasting base from which 36 forecasts were generated at different lead times and then comparisons were made with the actual data-points and the forecast errors were computed and analyzed. The demand patterns created were:

Poisson. A Poisson process was generated by calling the Poisson function from the School of Systems and Logistics Time Sharing Library and generating 120 data-points. The mean was set at ten and the series was begun with the SEED (12345).

Linearly Increasing Poisson. The Poisson function was called and the mean was originally set at ten and then the mean was incremented by 1/12 with each data-point

TABLE 2.1

SIGNIFICANT VARIABLES

Description of Variable	Data Source	Units of Data Measurement	Value Level	Measurement Scale
Total OFM Demand Rate	D041 Forecast	To 4 Decimal Points	Discrete Infinite	Ratio
Base NRTS Rate	D041 Forecast	Nearest Percent	Discrete Infinite	Ratio
Base Condemnation Rate	D041 Forecast	Nearest Percent	Discrete Infinite	Ratio
Depot Condemnation Rate	D041 Forecast	Nearest Percent	Discrete Infinite	Ratio
OFM Depot Demand Rate	D041 Forecast	To 4 Deminal Points	Discrete Infinite	Ratio
The Flying Program	D041 Forecast	Nearest Hour	Discrete Infinite	Ratio

generation. The mean of the 120th data-point was 20. The series was begun with the SEED (12345).

Linearly Decreasing Poisson. The Poisson function was called and the mean was set at 40 and then the mean was decremented by 1/6 with each data-point generation. The mean of the 120th data-point was 20. The series was begun with the SEED (12345).

Alternating Linear Poisson. The Poisson function was called and the mean was originally set at 20 and then the mean was incremented by 1/3 for 12 data-point generations and then decremented by 1/3 for 6 data-point generations and this process continued until 120 data-points were generated.

Sine. The sine and Poisson functions were called and the mean of the Poisson was set at 10 and then allowed to vary according to a sine function of amplitude 20. In order to eliminate negative values, this oscillating function was shifted up by adding a constant of value 50 to it. The series was begun with the SEED (12345) and 120 data-points were generated.

Research Approach

The purpose of this research was to analyze the impact that errors in the depot reparable generations forecasts might have upon the scheduling of actual production

through the depot facility. The basic approach was to: (1) describe the underlying forecasting model used by the D041 system to predict depot reparable generations; (2) compare the D041 forecasts of a limited number of depot reparable items with the actual number for the same periods; and (3) use time series analysis techniques to forecast from the same data base and to compare the results.

Testing the First Research Hypothesis

The first research hypothesis is: the expected value of the distribution of the forecast errors presently found in the D041 system is not zero, i.e., that the forecast is biased. For example, if e_τ represents forecast errors over a forecast horizon of length τ, then $E(e_\tau) \neq 0$, $\tau = 1,2,3,\ldots;$ if the forecast is unbiased.

The statistical hypothesis can be formulated as:

$$H_0: \; E(e_\tau) = 0$$
$$H_1: \; E(e_\tau) \neq 0$$

Since each forecast is an estimator of the actual reparable generations, the symbol used in this research for the forecast of g(t) made τ periods earlier is $\hat{g}_\tau(t)$. The forecast error term $e_\tau(t)$ at time t is defined as the difference between the forecast value for time t made at time $t-\tau$ and the actual value observed at time t. It is computed as: $e_\tau(t) = \hat{g}_\tau(t) - \hat{g}(t)$ for some fixed lead time τ. The $e_\tau(t)$,

$t = 1,2,3,\ldots$ will yield a distribution of forecast errors for the fixed lead time τ.

To facilitate understanding, imagine that time has been stopped at the beginning of the first quarter FY73. Using the single moving average method at this fixed point in time, the forecast mechanism can compute the forecast for every time point in the future (i.e., for any lead time τ), using the previous 24 months of actual generations. Since this fixed point in time has passed, the actual number of generations is available for comparison so that the $e_\tau(t)$ can be computed (see Table 2.2).

The expected value of the forecast error for lead time τ is computed as follows:

$$E(e_\tau) = \frac{\sum_{t=1}^{n} [\hat{g}_\tau(t) - g(t)]}{n}. \qquad (2.1)$$

This is the expected value for the error distribution associated with a fixed lead time, τ. If the lead time is changed, another forecast error distribution and its mean can be generated.

Lead times of 1, 3, 6, 9, 12, 15, 18, 24, 27, 30, 33, and 36 months were used in this study and the forecasting data base was fixed at 24 data points. Thirty-six forecasts were computed for each lead time and these 36 forecasts yielded 36 forecast errors and can be approximated by a normal distribution by virtue of the Central

TABLE 2.2

EXAMPLE DETERMINATION OF FORECAST ERRORS FOR $\tau=1$ AND $\tau=2$

Lead Time τ	Month						
	1	2	3	4	5	6	7
1	$\hat{g}_1(2)$	$\hat{g}_1(3)$	$\hat{g}_1(4)$	$\hat{g}_1(5)$	$\hat{g}_1(6)$	$\hat{g}_1(7)$	$\hat{g}_1(3)$
		$g(2)$	$g(3)$	$g(4)$	$g(5)$	$g(6)$	$g(7)$
2	$\hat{g}_2(3)$	$\hat{g}_2(4)$	$\hat{g}_2(5)$	$\hat{g}_2(6)$	$\hat{g}_2(7)$	$\hat{g}_2(8)$	$\hat{g}_2(9)$
			$g(3)$	$g(4)$	$g(5)$	$g(6)$	$g(7)$

NOTE: $\hat{g}_\tau(t)$ is the τ-period forecast for the month t made in month $t-\tau$.

$g(t)$ is the actual number of generations for month t.

The one-month forecast error for month 2 is $e_1(2)-\hat{g}_1(2)-g(2)$.

The two-month forecast error for month 3 is $e_2(3)=\hat{g}_2(3)-g(3)$.

The expected forecast error for lead time τ is then given by:

$$\frac{1}{t-\tau}\sum_{t=\tau}^{T} e_\tau(t).$$

Limit Theorem. The research hypothesis was designed to determine whether the forecasting method used in the D041 system was biased. The risk of making a Type I error in the statistical test, i.e., the probability of rejecting a true hypothesis, should be minimized. The level of significance (α) was set at .01 with the resulting two-tailed test having a rejection region of .005 in each tail. The hypothesis test was applied to each of the forecast error distributions. If the statistical test yielded the rejection of the null hypohtesis, the inference to be made was that the D041 forecast was biased for the forecast horizon tested because the probability of observing an error of that magnitude would be .01 or less if the hypothesis was true.

A computer program was written to compute the forecast errors and to make the statistical test for the D041 forecasting method (see Appendix A).

Testing the Second Research Hypothesis

The second research hypothesis states: Other forecasting methods may exhibit less bias than the D041 forecasting method. If the D041 forecasting method was shown to be biased for items whose reparable generations followed a particular pattern, then time series analysis forecasting methods were used to determine if an appropriate model existed which would exhibit less bias.

The same demand patterns which resulted in biased forecasts using the D041 forecasting method were used to make time series analysis forecasts. The same data bases and lead time periods were used. The forecast errors for each forecast were generated and e_τ for each lead time was computed. After $E(e_\tau)$ for each lead time had been computed, the same hypothesis test used in evaluating the D041 forecasting method was used, i.e.,

$$H_0: \quad E(e_\tau) = 0$$

$$H_1: \quad E(e_\tau) \neq 0.$$

The alpha level was again set at .01 with the resulting two-tailed test having a rejection region of .005 in each tail. If the statistical test yielded the rejection of the null hypothesis, the inference to be made was that the time series analysis forecast was biased because the probability of observing an error of that magnitude would be .01 or less, if the null hypothesis was true. If the null hypohtesis cannot be rejected, then the inference is that the time series analysis forecast exhibits less bias than the D041 forecasting method.

Time Series Analysis

A general discussion of time series analysis is presented in this section, with a more mathematical approach presented in Appendix B.

A time series is a set of observations taken at equally spaced time intervals, i.e., G_t, $t=1,2,3,\ldots,n$. As opposed to the usual assumption of stochastic independence of observations found in other forecasting techniques, time series models specifically assume that the observations are correlated. Box and Jenkins have developed an iterative method for modelling this dependence among the observations in a time series (1). Rather than being a model fitting process, it is a model building process, with the model determined upon the basis of data analysis rather than upon assumption.

Determination of a model is accomplished in a stage called identification. This stage is followed by parameter estimation, and then by a series of diagnostic checks to determine if the model identified provides an adequate description of the stochastic process generating the data. If in the checking phase the model is shown to be deficient in some way, then the identification phase is reentered and the entire process is repeated. When a model has been adequately identified, then it is used to forecast future realizations of this stochastic process.

The basic tools for identification of the time series G_t are the autocorrelation and partial autocorrelation functions of the observed data (1:147). The general time series model can be described in the following manner. First, a backshift operator is defined such that

$Bg_t = g_{t-1}$. The symbol a_t is used to denote a random error, entering the model in period t. These errors are assumed to be independent, normally distributed random variables with $E(a_t)=0$, for $t=1,2,3,\ldots,n$ and a constant variance σ_a^2. Using these definitions and disregarding seasonal and trend factors, the most general form of the Box-Jenkins model has an "autoregressive-intergrated-moving average" (ARIMA) form:

$$(1-\phi_1 B-\phi_2 B^2-\ldots.\phi_p B^p)(1-B)^d \tilde{G}_t$$

$$= (1-\theta_1 B-\theta_2 B^2-\ldots-\theta_q B^q)a_t \qquad (2.2)$$

where:

$$\tilde{G}_t = \begin{cases} G_t \text{ if } d > 0 \\ \\ G_t-\mu \text{ if } d=0 \end{cases}$$

μ = the series mean

G_t = values of g at time t

a_t = error term

$\phi_m, m = 1,2,\ldots,p$ are autoregressive parameters and appear in the autoregressive factor in the model.

$\theta_m, m = 1,2,\ldots,q$ are moving average parmeters and appear in the moving average factor in the model.

The model is based on the assumption that the series G_t is stationary. If the series is not stationary, then its first, second, etc. differences are investigated for stationarity, i.e., by checking $(1-B)^d\hat{G}(t)$ for various values of d. Shorter notation for the complete model is ARIMA (p,d,q), where p refers to the order of autoregressive factors, d refers to the order of differencing, and q refers to the order of the moving average factors.

Two very important concepts, stationarity and differencing, have been introduced and require further explanation. A process is considered stationary if the joint distribution of the observations does not vary with time;

$$p(g_t,\ldots,g_{t+\tau}) = p(g_{t+m},\ldots,g_{t+\tau+m}) \qquad (2.3)$$

where p (·) is the joint probability density function. If the process is stationary, the expected value of the observations, $E(g_t)$ does not vary with time, i.e., $E(g_t) = E(g_{t+\tau})$

Further, the covariance between any two observations from a stationary process depends only upon the number of time period (τ) separating the two observations, i.e.,

$$C(g_t,g_{t+\tau}) = C(g_{t+m},g_{t+\tau+m}). \qquad (2.4)$$

In reality, many time series are not stationary. Nonstationary time series may have a mean which is itself a function of time (1:7,85,92). However, such series often exhibit what is called homogeneous nonstationarity, where the behavior of the series is similar at different points in time. If a series is stationary in a homogeneous sense, the successive changes, or differences, of that series are stationary (6:56). Thus, a nonstationary time series can often be analyzed by finding a difference of the series which is stationary and applying stationary techniques to it (6:56-58).

Usually, it is necessary to take only the first or second differences of a series to achieve stationarity (1:175). The first difference of a series, Δ_t, is defined to be $\Delta_t(g)=g_t-g_{t-1}$. The second differences are the differences of the first differences and are given by:

$$\Delta_t^2(g) = \Delta_t(g)-\Delta_{t-1}(g) = (g_t g_{t-1})-(g_{t-1}-g_{t-2})$$

$$= g_t-2g_{t-1}+g_{t-2} \qquad (2.5)$$

The computer programs for building the Box and Jenkins models for the analysis and forecasting of time series are available on the CREATE system (see Appendix D). A computer program that will generate data files from random generators or accept data entered from the terminal

to be read into the Box and Jenkins programs has been developed and is included in Appendix C.

List of Assumptions

1. The D041 forecasting procedures have been unchanged and have been consistently employed during the period analyzed by this research.

2. The distribution of forecast errors follows a normal distribution.

3. The $e_\tau(t)$ terms, for a set lead time τ are independent.

4. The a_t terms are independent, with an expected value of 0, and a constant variance σ_a^2.

List of Limitations

1. The results of this research will be limited to the forecasting methods (principally time series models) applied to the reparable generations of items which follow the particular distribution patterns discussed.

2. The guide for the development of data files for input to the Box and Jenkins models and for the estimation and identification of time series models are applicable only to the computer programs on the CREATE system. Slight modifications may be required to adapt the programs to other systems and computer languages.

CHAPTER III

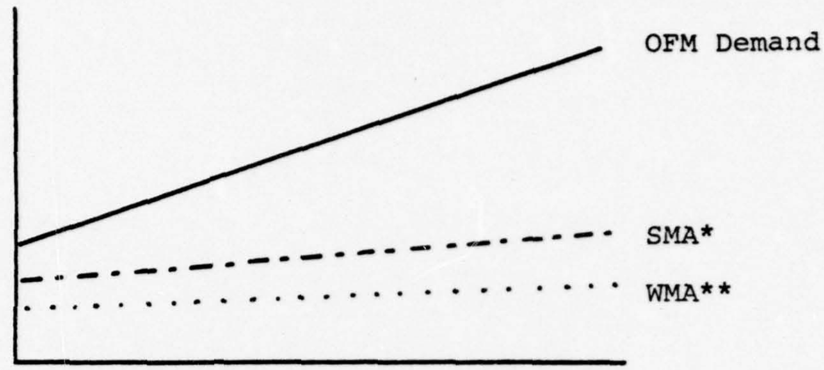
FINDINGS AND ANALYSIS

The D041 Forecasting Method

The D041 system uses a single moving average forecasting method based on the most recent eight quarters of data as its basic forecasting method (2). In computing the OFM Demand Rate, which is used in conjunction with future programs to determine projected reparable generations, the D041 system uses a weighted single moving average forecasting method (2). The weight depends on the changes in the program and OFM rates.

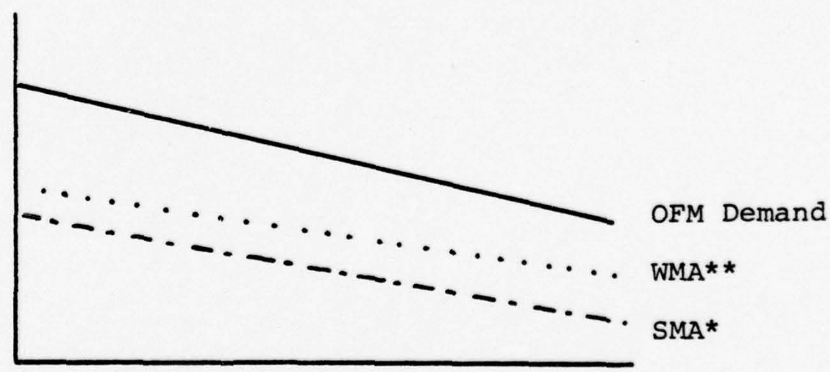
Consequently, when the quarterly programs follow a decreasing trend (see Figure 3.1) the currently computed OFM demand rate will be less responsive to these trends in the quarterly rates than will the single moving average of the quarterly rates. Also, if the quarterly OFM demand rates follow an increasing trend (Condition 1 on Figure 3.1), the computed demand rate will be smaller than the single moving average of the rates, and if the quarterly OFM demand rates follow a decreasing trend (Condition 2 on Figure 3.1), the computed demand rate will be larger than the single moving average.

Similarly, when the quarterly programs follow an increasing trend (see Figure 3.2), the currently computed



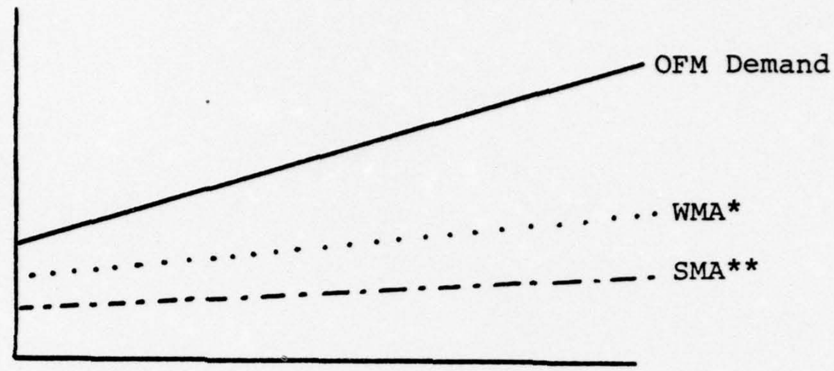
Condition 1
OFM Demand Rate Increasing

*SMA=Single Moving Average
**Weighted Moving Average



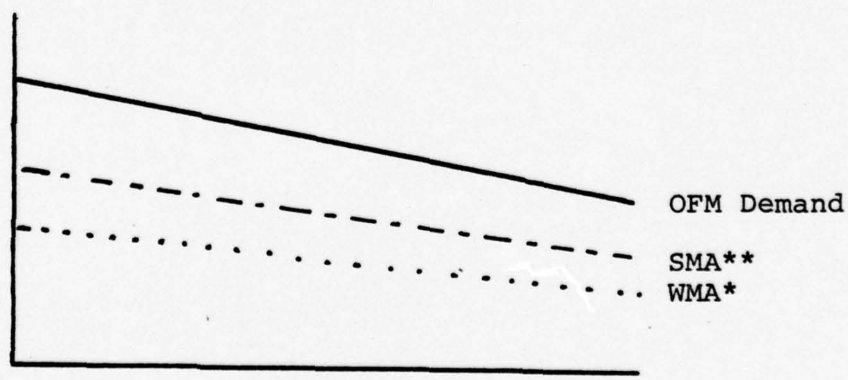
Condition 2
OFM Demand Rate Decreasing

Fig. 3.1. Decreasing Program



Condition 3
OFM Demand Rate Inceasing

*WMA=Weighted Moving Average
**SMA=Single Moving Average



Condition 4
OFM Demand Rate Decreasing

Fig. 3.2. Increasing Program

OFM demand rate will be more responsive to any trends in the quarterly rates than the single moving average of the quarterly rates. Thus, if the quarterly OFM demand rates follow an increasing trend (Condition 3 on Figure 3.2), the computed demand rate will be larger than the single moving average of the rates, and if the quarterly rates follow a decreasing trend (Condition 4 on Figure 3.3), the computed demand rate will be smaller than the single moving average of the rates.

In the situation where the program is approximately constant over time, the OFM demand rate is essentially computed using a single moving average.

The equation for a single moving average method using "x" for the length of the base period, "t" for the period, and "G" for the variable would be:

$$SMA_t = \frac{1}{x} \sum_{n=t_0}^{t} G_n \qquad (3.1)$$

where:

$t_0 = t-(x-1)$.

The equation for a weighted single moving average using "W" for the weighted value would be:

$$WSMA_t = \frac{1}{x}\sum_{n=t_0}^{t} W_n \tag{3.2}$$

where:

$t_0 = t-(x-1)$

The D041 forecasting method was investigated to determine if the forecasts were biased. A single moving average forecasting model was computerized (Appendix A) and was used to compute the bias of forecasts using five computer-generated data patterns. The forecast was initialized on the first 24 data points of each pattern and then 36 forecasts were made incrementing the data base by one for each data pattern and for each of the 13 different lead times. This method yielded 36 forecast errors for each lead time and these forecast errors were used to determine the bias and statistical significance of the bias for each data pattern. Lead times of 1, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, and 36 months were used in this study.

The results of this analysis are presented below for each pattern tested.

Poisson Pattern

As expected, forecasts of data from this pattern were found to be statistically unbiased for each lead time (refer to Tables F.1 and F.2).

Linearly Increasing
Poisson Pattern

The single moving average forecasts were found to be statistically biased at an alpha level of .01 for lead times of 6 months of greater. For the lead time of 3 months, the forecast was statistically biased at an alpha level of .05 and at a lead time of 1 month was statistically biased at alpha level of .10 (refer to Tables F.3 and F.4).

Linearly Decreasing
Poisson Pattern

The results were essentially the same as those of the linearly increasing Poisson pattern except the forecasts were more biased because a steeper slope was used for this pattern. The forecasts were statistically biased for all lead times (refer to Tables F.5 and F.6).

Alternating Linear Increasing
and Decreasing Poisson Pattern

The single moving average forecasts were found to be statistically biased at lead time of 12 months or greater at an alpha level of .01. At lead times of one

to 9 months the forecasts were statistically biased at an alpha level of .10 (refer to Tables F.7 and F.8).

Sine Pattern

The sine pattern was found to be statistically unbiased at an alpha level of .01, but was found to be biased for all lead times at an alpha level of .20 (refer to Tables F.9 and F.10).

Time Series Analysis Forecasting Results

This section contains the results from the time series analysis forecasting techniques. The single moving average forecasting method was shown to be unbiased at an alpha level of .01 for the Poisson and Sine patterns and consequently time series analysis forecasts were not made for these data patterns. Time series analysis methods were used to make forecasts for the linearly increasing Poisson, linearly decreasing Poisson, and alternating linear Poisson patterns. The initial 24 data points were used as the forecasting base for each data pattern. From that base, forecasts were made 96 time periods into the future. Thirty-six forecast errors were computed for each lead time and the same statistical tests used in determining the bias of the single moving average forecasts were used to determine the bias of the time series analysis forecasts. The time series analysis

forecasts were found to be unbiased for each lead time with each data pattern at an alpha level of .01 (refer to Appendix F and Tables F.11 through F.13.)

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The first research hypothesis stated that the expected value of the distribution of forecast errors presently found in the D041 system was not zero, i.e., the forecast was biased. The research concluded that reparable generations which followed a Poisson process with a constant mean or one in which the mean followed a sine distribution could be forecast without bias by using the D041 single moving average forecasting method. The research concluded that the single moving average forecasting method produced biased forecasts of reparable generations of recoverable items when the reparable generations followed a linearly increasing Poisson, a linearly decreasing Poisson, or an alternating linear Poisson pattern.

The second research hypothesis stated that other predictive methods might exhibit less bias than the D041 forecasting method. The research concluded that time series analysis techniques could be used to produce unbiased forecasts for linearly increasing, linearly decreasing and for alternating linear Poisson processes.

Recommendations

Recommendations for further research include the following:

1. Inasmuch as this thesis was based on data generated following specific random processes, future studies should be accomplished using representative actual data.

2. Since a number of random processes were investigated in this thesis, a future study could be accomplished to determine if particular federal stock classes of recoverable items can be identified to follow any of these patterns.

3. An investigation of different error criteria such as the mean absolute error, average absolute difference, and/or relative forecast error to determine the bias of the time series analysis and single moving average forecasting methods should be performed.

4. An examination should be made of the negotiation process to determine the differences between forecasted reparable generations and the number of units negotiated to be put in repair for the same time period.

5. An examination should be made of the productive process with emphasis on the accuracy of the methods used in forecasting availability of manhours, facility time, and spare parts in the maintenance system. A comparison could be made using the methodology of this

research effort to determine the accuracy by comparing the forecasted availability with actual availability. If discrepancies are found, other forecasting methodologies, particularly time series anlaysis should be investigated further, to determine if better forecasts can be made.

APPENDIXES

APPENDIX A

FORECAST BIAS

APPENDIX A

FORECAST BIAS

The program contained in this appendix was designed to test the bias of the single moving average and other forecasting methods such as the time series analysis method. The program requires a minimum of 60 data observations and can handle up to 200 data observations. The first 24 data points are as used in the initial forecasting base and then 36 forecasts are computed incrementing the forecasting base by one for each new forecast. The error terms can then be computed at different lead times and the bias can be determined as these lead times. The program also computes a statistic which can be compared with t distribution values to determine the alpha level of significance of the bias. Data observations can be entered directly from the terminal or the random processes which are built into the program can be used to generate observations.

It should be noted that if a lead time of 36 is being investigated, at least 96 observations would have to be initially entered into the program. The program uses the first 24 as the initial forecasting base, then computes 36 forecasts which requires up to and including the 60th

data point, and then finally another 36 observations are required to compute 36 forecast errors which require up to and including the 96th data point. If insufficient data points are entered, the program will use zeroes for the required remaining data points and the results will obviously be inaccurate.

The computer program was written in the FORTRAN IV computer language and can be run as follows:

1. The program asks for the number of observations. These observations can be entered from the terminal or from the built-in random processes.

2. The program asks if single moving average forecasts are desired or if the bias of another forecasting method is being investigated. If single moving average forecasts are desired, then the program automatically computes 36 single moving average forecasts initializing on the first 24 data points. Lead times can then be entered and the program computes the bias and test statistic for the entered lead times.

3. If testing the bias of another forecasting method, lead times can be entered and the bias and test statistic for these lead times will be computed by the program.

The following random processes were built into the program:

1. A Poisson process with a mean of ten.

2. A Poisson process with a linearly increasing mean. The mean is initially set at 10 and then is incremented by one-twelfth for each newly generated data point.

3. A Poisson process with a linearly decreasing mean. The mean is initially set at 40 and then is decremented by one-sixth for each newly generated data point.

4. A Poisson process with an alterating linear mean. The mean increases for the first 12 data points and then decreases for the next 6 data points and then continues to increase and decrease in the same manner to the 120th data point and then increases from this point to the 200th data point.

5. A process whose values are the sine function of the output of a Poisson process.

The complete computer program comprises the remainder of this appendix.

```
10*#RUN *=(ULIB)GRADLIB/TSS,R
15 PRINT,"ENTER THE NUMBER OF OBSERVATIONS"
16 READ,NOF
20 DIMENSION A(200),SMA(36),XE(36)
30 REAL TIME(200),XXE(36)
40 PRINT,"IF ENTERING DATA FROM TERMINAL ENTER 0 IF ENTERING DATA"
50 PRINT,"FROM RANDOM GENERATOR ENTER 1"
60 PRINT," "
70 PRINT," "
80 READ,KKK
90 IF(KKK.EQ.1)GO TO 40
100 PRINT,"     ENTER THE OBSERVATIONS"
110 READ,(A(I),I=1,NOF)
120 PRINT," "
130 PRINT," "
140 GO TO 190
150 40 CONTINUE
160 X=RND(12345)
170 PRINT,"ENTER 1 FOR POISSON DISTRIBUTION 2 FOR INCREASING LINEAR"
180 PRINT,"POISSON 3 FOR DECREASING LINEAR POISSON 4 FOR RANDOM LINEAR"
190 PRINT," POISSON 5 FOR SINE 6 FOR EXPONENTIAL AND 7 FOR"
200 PRINT,"HYPERGEOMETRIC DISTRIBUTION"
210 PRINT," "
220 READ,IRG
230 GO TO(1,3,5,90,11,13,19),IRG
240 1 DO 2 I=1,NOF
250 A(I)=POISSON(10.,1.)
260 2 CONTINUE
270 GO TO 190
280 3 DO 6 I=1,NOF
290 A(I)=POISSON(10+I/12.,1.)
300 6 CONTINUE
310 GO TO 190
320 5 DO 7 I=1,NOF
```

```
330 A(I)=POISSON(40-I/6.,1.)
340 7 CONTINUE
350 GO TO 190
360 90 DO 8 I=1,12
370 A(I)=POISSON(20+I/3.,1.)
380 8 CONTINUE
390 DO 101 I=13,18
400 A(I)=POISSON(24-(I-12)/3.,1.)
410 101 CONTINUE
420 DO 102 I=19,30
430 A(I)=POISSON(22+(I-18)/3.,1.)
440 102 CONTINUE
450 DO 103 I=31,36
460 A(I)=POISSON(26-(I-30)/3.,1.)
470 103 CONTINUE
480 DO 104 I=37,48
490 A(I)=POISSON(24+(I-36)/3.,1.)
500 104 CONTINUE
510 DO 105 I=49,54
520 A(I)=POISSON(28-(I-48)/3.,1.)
530 105 CONTINUE
540 DO 106 I=55,66
550 A(I)=POISSON(26+(I-54)/3.,1.)
560 106 CONTINUE
570 DO 107 I=67,72
580 A(I)=POISSON(30-(I-66)/3.,1.)
590 107 CONTINUE
600 DO 108 I=73,84
610 A(I)=POISSON(28+(I-72)/3.,1.)
620 108 CONTINUE
630 DO 109 I=85,90
640 A(I)=POISSON(32-(I-84)/3.,1.)
650 109 CONTINUE
660 DO 111 I=91,102
```

```
670 A(I)=POISSON(30+(I-90)/3.,1.)
680 111 CONTINUE
690 DO 112 I=103,108
700 A(I)=POISSON(34-(I-102)/3.,1.)
710 112 CONTINUE
720 DO 113 I=109,NOF
730 A(I)=POISSON(32+(I-108)/3.,1.)
740 113 CONTINUE
750 GO TO 190
760 11 DO 9 I=1,NOF
770 A(I)=20*SIN(POISSON(10.,1.))+50
780 9 CONTINUE
790 GO TO 190
800 13 DO 17 I=1,NOF
810 A(I)=EXPONT(5.)
820 17 CONTINUE
830 GO TO 190
840 19 DO 21 I=1,NOF
850 A(I)=HYPERG(5,.10,100.)
860 21 CONTINUE
870 GO TO 190
880 191 PRINT,"IF MAKING STATISTICAL COMPARISON OF ANOTHER FORECAST"
890 PRINT,"WITH A SINGLE MOVING AVERAGE FORECAST ENTER 1 OTHERWISE 0"
900 READ,KKKK
910 IF(KKKK.EQ.1)GO TO 192
920 DO 10 I=1,36
930 DO 20 J=I,23+I
940 SMA(I)=SMA(I)+A(J)
950 20 CONTINUE
960 10 CONTINUE
970 DO 30 I=1,36
980 SMA(I)=SMA(I)/24
990 30 CONTINUE
1000 PRINT," "
```

```
1010 PRINT," "
1020 PRINT," "
1030 PRINT,"          36 SINGLE MOVING AVERAGE FORECASTS"
1040 PRINT,"          BASED ON A 24 POINT DATA BASE"
1050 PRINT," "
1060 PRINT," "
1070 PRINT," "
1080 PRINT 200,(SMA(I),I=1,36)
1090 200 FORMAT(2X,F8.4,3X,F8.4,3X,F8.4,3X,F8.4,3X,F8.4,3X,F8.4)
1100 PRINT," "
1110 PRINT," "
1120 100 PRINT,"          ENTER LEAD TIME"
1130 XSUM=0
1140 AVEXR=0
1150 SXIS=0
1160 STND=0
1170 TEST=0
1180 PRINT," "
1190 PRINT," "
1200 PRINT," "
1210 READ,B
1220 DO 60 L=1,36
1230 XE(L)=SMA(L)-A(23+B+L)
1240 60 CONTINUE
1250 DO 70 L=1,36
1260 XSUM=XSUM+XE(L)
1270 70 CONTINUE
1280 AVEXR=XSUM/36
1290 DO 80 L=1,36
1300 SXIS=SXIS+(XE(L)**2)
1310 80 CONTINUE
1320 STND=((SXIS-(36*(AVEXR**2)))/35)**0.50
1330 TEST=(AVEXR*6)/STND
1340 PRINT," "
```

```
1350 PRINT," "
1360 PRINT,"          ERROR SUM         BIAS          STND DEV         TEST"
1370 PRINT," "
1380 PRINT 300,XSUM,AVEXR,STND,TEST
1390 300 FORMAT(8X,F10.4,6X,F9.4,7X,F8.4,8X,F8.4)
1400 PRINT," "
1410 PRINT," "
1420 PRINT,"   IF YOU WANT ANOTHER LEAD TIME ENTER 1 IF NOT ENTER 0"
1430 PRINT," "
1440 PRINT," "
1450 READ,K
1460 IF (K.EQ.1)GO TO 100
1470 GO TO 770
1480 190 CONTINUE
1490 PRINT," "
1500 PRINT," "
1510 PRINT,"ENTER 1 IF YOU WANT TO CHECK INPUT DATA ENTER 0 OTHERWISE"
1520  PRINT," "
1530 PRINT," "
1540 READ, KK
1550 PRINT," "
1560 PRINT," "
1570 IF(KK.EQ.0)GO TO 191
1580 PRINT,"          RANDOM VARIABLE PRINTOUT"
1590 PRINT," "
1600 PRINT," "
1610 PRINT 500,(A(I),I=1,NOF)
1620 500 FORMAT(2X,F4.0,2X,F4.0,2X,F4.0,2X,F4.0,2X,F4.0,2X,F4.0,2X,F4.0,2X,F4.0,
1630& 2X,F4.0,2X,F4.0,2X,F4.0,2X,F4.0,/)
1640 GO TO 191
1650 192 PRINT,"ENTER THE NUMBER OF FORECASTED OBSERVATIONS"
1660 READ,NOB
1670 PRINT,"ENTER THE FORECASTED OBSERVATIONS"
1680 READ,(TIME(I),I=1,NOB)
```

```
1690 PRINT,"            FORECASTED OBSERVATIONS"
1700 PRINT," "
1710 PRINT," "
1720 PRINT 200,(TIME(I),I=1,NOB)
1730 1000 PRINT,"ENTER LEAD TIME"
1740 XXSUM=0
1750 XAVER=0
1760 XSXIS=0
1770 XSTND=0
1780 XTEST=0
1790 READ,BB
1800 DO 600 I=1,36
1810 XXE(I)=TIME(I+BB-1)-A(23+BB+I)
1820 600 CONTINUE
1830 DO 700 I=1,36
1840 XXSUM=XXSUM+XXE(I)
1850 700 CONTINUE
1860 XAVER=XXSUM/36
1870 DO 800 I=1,36
1880 XSXIS=XSXIS+(XXE(I)**2)
1890 800 CONTINUE
1900 XSTND=((XSXIS-(36*(XAVER**2)))/35)**0.50
1910 XTEST=(XAVER*6)/XSTND
1920 PRINT,"            ERROR SUM       BIAS          STND DEV          TEST"
1930 PRINT 300,XXSUM,XAVER,XSTND,XTEST
1940 PRINT,"IF YOU WANT ANOTHER LEAD TIME ENTER 1 IF NOT ENTER 0"
1950 READ, NNNN
1960 IF (NNNN.EQ.1)GO TO 1000
1970 770 STOP
1980 END
```

APPENDIX B

MATHEMATICAL DESCRIPTION OF TIME SERIES ANALYSIS MODELS

APPENDIX B

MATHEMATICAL DESCRIPTION OF TIME SERIES ANALYSIS MODELS

In Chapter II a time series was defined to be a set of observations taken at equally spaced time intervals, e.g., g_t, $t=1,2,\ldots,n$. A backshift operatior B was defined by the relationship $Bg_t=g_{t-1}$ and $B^m g_t=g_{t-m}$. A forward shift operator F can be defined as the inverse of B ($F=B^{-1}$), e.g., $Fg_t=g_{t+1}$ and $F^m g_t=g_{t+m}$.

Another operator introduced in Chapter II was the backward difference operator Δ where

$$\Delta g_t = g_t - g_{t-1} = (1-B)g_t \; . \tag{B.1}$$

White noise, denoted as a_t, was defined as a series of independent random shocks. A general stochastic process, g(t), can be modeled as a linear combination of these random shocks. This combination is often called a linear filter and is described pictorially below:

$$a_t \longrightarrow \overset{\psi(B)}{\boxed{\text{Linear Filter}}} \longrightarrow g_t \tag{B.2}$$

The linear filtering operation simply takes a weighted sum of previous observations so that:

$$g(t) = \mu+a_t+\psi_1 a_{t-1}+\psi_2 a_{t-2}+\ldots \quad \text{(B.3)}$$

$$= \mu+\psi(B)a_t$$

In general, μ is a parameter that determines the level of the process and $\psi(B)=1+\psi_1 B+\psi_2 B^2+ \ldots$ is the linear operator which transforms the sequence $a_t, a_{t-1}, \ldots,$ into g_t and is called the transfer function of the filter.

Autoregression Models

A model which can be extremely useful in the representation of certain stochastic processes is the autoregressive model. In this model, the current value of the process is expressed as a finite, linear combination of previous values of the process and a shock a_t. Denoting the values of a process by $g_t, g_{t-1}, g_{t-2}, \ldots$ and letting $\hat{g}_t, \hat{g}_{t-1}, \hat{g}_{t-2}, \ldots$ be deviations of g from μ (i.e., $\hat{g}_t=g_t-\mu$), then:

$$\hat{g}_t = \phi_1 \hat{g}_{t-1}+\phi_2 \hat{g}_{t-2}+ \ldots +\phi_p \hat{g}_{t-p}+a_t \quad \text{(B.4)}$$

is called an autoregressive (AR) process of order p. The reason for this name is that a linear model $g=\phi_1 \hat{X}_1+ \phi_2 \hat{X}_2+ \ldots +\phi_p \hat{X}_p+a$ relating a dependent variable g to a set of independent variables, $X_1, X_2, \ldots, X_p$ plus an error a is referred to as a regression model, that is g is said

to be regressed on $X_1, X_2, \ldots, X_p$. In (B.4) the variable g is regressed on previous values of itself; hence the model is autoregressive. An autoregressive operator of order p can be defined as:

$$\phi(B) = 1-\phi_1 B-\phi_2 B^2- \ldots -\phi_p B^p . \quad \text{(B.5)}$$

The autoregressive model may then be written concisely as: $\phi(B)\hat{g}=a_t$. The model contains p+2 unknown parameters, $\mu, \phi_1, \phi_2, \ldots, \phi_p, \sigma_a^2$ which, in practice, have to be estimated from the data and σ_a^2 is the variance of the white noise a_t.

The autoregressive model is a special case of the linear filter model (B.4). For example, eliminate $\hat{g}_{t-1}$ from the right hand side of (B.4) by substituting $\hat{g}_{t-1}=\phi_1\hat{g}_{t-2}+\phi_2\hat{g}_{t-3}+ \ldots +\phi_p\hat{g}_{t-p-1}+a_{t-1}$. Likewise substitute for $\hat{g}_{t-2}$, and so on, yielding an infinite series in the a's. Symbolically, $\phi(B)\hat{g}_t=a_t$, which is equivalent to $g_t=\psi(B)a_t$ with $\psi(B)=\phi^{-1}(B)$. An autoregressive process may or may not be stationary. For the process to be stationary, the ϕ's must be chosen so that the weights $\psi_1, \psi_2, \ldots$ in the $\psi(B)=\phi^{-1}(B)$ form a convergent series.

Moving Average Models

The autoregressive model (B.4) expresses the deviation on the process $\hat{g}_t$ as a finite weighted sum of p previous deviations $\hat{g}_{t-1}, \hat{g}_{t-2}, \ldots, \hat{g}_{t-p}$ plus a random shock

a_t. Equivalently it expresses $\hat{g}_t$ as an infinite weighted sum of a's. Another kind of model with great practical importance in representing observed time series is the finite moving average process (1:10). In this model $\hat{g}_t$ is linearly dependent on a finite number (q) of previous a's. Thus:

$$g_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \dots - \theta_q a_{t-q} \tag{B.6}$$

is called a moving average (MA) process of order q. The name "moving average" is somewhat misleading since the weights $1, -\theta_1, -\theta_2 - \dots -\theta_q$ need not total unity nor need they be positive. If a moving average operator of order q is defined as $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$, then the moving average model can be written concisely as:

$$g_t = \theta(B) a_t \ ,$$

which contains q+2 unknown parameters $\mu, \theta_1, \dots, \theta_q, \sigma_a^2$ which, in practice, are estimated from the data.

<u>Mixed Autoregressive-Moving Average Models</u>

In order to achieve greater flexibility in the fitting of actual time series, it is advantageous to include both autoregressive and moving average terms in the model.

This leads to the general mixed autogressive moving average (ARMA) model:

$$g_t = \phi_1 g_{t-1} + \dots + \phi_p g_{t-p} + a_t - \theta_1 a_{t-1} - \dots \theta_q a_{t-q}$$

or (B.7)

$$\phi(B) g_t = \theta(B) a_t$$

which employs p+q+2 unknown parameters, $\mu, \phi_1, \dots, \phi_p, \theta_1, \dots, \theta_q, \sigma_a^2$, which are estimated from the data.

Nonstationary Models

Many time series encountered in practice exhibit nonstationary behavior, i.e., they do not vary around a fixed mean (1:11). Such series may nevertheless exhibit homogeneous behavior of a kind. In particular, although the general level about which fluctuations are occurring may be different at different times, the broad behavior of the series, when differences in level are allowed for, may be similar. Such behavior may be represented by a generalized autoregressive operator Q(B), in which one or more of the zeros of the polynomial Q(B), i.e., one or more of the roots of the equation Q(B)=0, is unity. Thus, the operator Q(B) can be written: $Q(B)=\phi(B)(1-B)^{\alpha}$, where $\phi(B)$ is the stationary operator. The general model,

which can represent homogeneous nonstationary behavior is of the form:

$$Q(B)g_t = \phi(B)(1-B)^d g_t = \theta(B)a_t$$

that is

$$\phi(B)w_t = \theta(B)a_t \tag{B.8}$$

where

$$w_t = \Delta^d g_t \tag{B.9}$$

Homogeneous nonstationary behavior can therefore be represented by a model which calls for the d'th difference of the process to be stationary. In practice d is usually 0,1, or 2 (1:11). The process, described by (B.8) and (B.9), provides a powerful model for describing stationary and nonstationary time series and is called an autoregressive integrated moving average (ARIMA) process of order (p,d,q). The process is defined by:

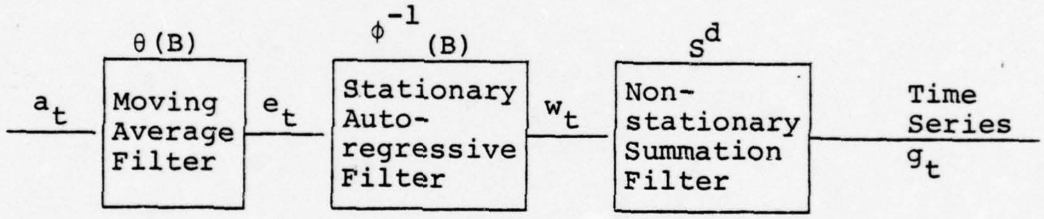
$$w_t = \phi_1 w_{t-1}+\ldots+\phi_p w_{t-p}+a_t-\theta_1 a_{t-1}-\ldots-\theta_q a_{t-q} \tag{B.10}$$

If w_t is replaced by $g_t-\mu$, when d=0, the model (B.10) includes the stationary mixed model (B.7) as a special

case and the pure moving average model (B.6). The word integrated is included in the ARIMA title because the relationship which is the inverse to (B.9) is: $z_t = S^d w_t$, where S is the summation operator defined by:

$$Sw_t = \sum_{b=0}^{\infty} w_{t-b} = w_t + w_{t-1} + w_{t-2} +, \ldots$$

Thus, the general ARIMA process may be generated from white noise a_t by means of three filtering processes as follows:



The first filter has input a_t, transfer function $\theta(B)$ and output e_t, where:

$$e_t = a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q} \qquad \text{(B.11)}$$

$$= \theta(B) a_t$$

The second filter has input e_t, transfer function $\phi^{-1}(B)$ and output w_t, where:

$$w_t = \phi_1 w_{t-1} + \dots + {}_p w_{t-p} + e_t \tag{B.12}$$

$$= \phi^{-1}(B) e_t$$

Finally, the third filter has input w_t and output g_t according to (B.10) and has transfer function S^d. For a detailed discussion of a special form of the model to employ representing seasonal time series see Box and Jenkins, Chapter 9.

Transfer Function Models

An important type of dynamic relationship between a continuous input and a continuous output is that in which deviations of input X and output Y, from appropriate mean values, are related by a linear differential equation of the form:

$$(1+\Xi_1 D+\dots+\Xi_R D^R)Y(t) = (H_0+H_1 D+\dots+H_s D^s)X(t-\tau) \tag{B.13}$$

where D is the differential operator d/dt, the Ξ's and Hs are unknown parameters, and τ is a parameter which measures the lead time or pure delay between input and output. A simple example of (B.13) would be a system where the rate of change in the output was proportional to the difference between input and output so that: $\Xi dY/dt=X-Y$ and hence $(1+\Xi D)Y=X$. Similarly, for discrete

data, the transfer function between an output Y and an input X, each measured at equispaced times, is given by the difference equation:

$$(1+\xi_1\Delta+\ldots+\xi_r\Delta^r)Y_t = (\eta_0+\eta_1\Delta+\ldots+\eta_s\Delta^s)X_{t-b} \qquad \text{(B.14)}$$

in which the differential operator D is replaced by the difference operator Δ. An expression of the form (B.14) containing only a few parameters ($r\leq 2$, $s\leq 2$), may often be used as an approximation to a dynamic relationship, whose true nature is more complex.

The linear model (B.14) may be written equivalently in terms of past values of the input and output by substituting $B=1-\Delta$ in (B.14), that is:

$$\begin{aligned}(1-\delta_1 B-\ldots-\delta_r B^r)Y_t &= (w_0-w_1B-\ldots-w_sB^s)X_{t-b} \\ &\qquad\qquad\qquad\qquad \text{(B.15)} \\ &= (w_0B^b-w_1B^{b+1}-\ldots-w_sB^{b+s})X_t\end{aligned}$$

or

$$\delta(B)Y_t = w(B)B^bX_t = \Omega(B)X_t$$

Alternatively stated, output Y_t and input X_t are linked by a linear filter:

$$Y_t = \gamma_0 X_t + \gamma_1 X_{t-1} + \gamma_2 X_{t-2} + \ldots \tag{B.16}$$

$$= \gamma(B) X_t$$

for which the transfer function

$$\gamma(B) = \gamma_0 + \gamma_1 B + \gamma_2 B^2 + \ldots \tag{B.17}$$

can be expressed as a ratio of two polynomials:

$$\gamma(B) = \Omega(B)/\delta(B) = \delta^{-1}(B)\Omega(B)$$

The linear filter is said to be stable if (B.17) converges for $|B| \leq 1$. The series of weights $\gamma_0, \gamma_1, \gamma_2, \ldots$, which appear in the transfer function (B.17) is called the inpulse response function. Note that for the model (B.14), the first b weights $\gamma_0, \gamma_1, \ldots \gamma_{b-1}$ are zero.

The transfer function (B.15) enables a reinterpretation of the stochastic models (B.6) and (B.7). The disturbance occurring in some output g will often have originated elsewhere in some variable with which g is dynamically linked by an equation of the form (B.14) (1:14). It might be expected that the complex stochastic behavior of a random variable g_t might be expressed in terms of another random variable a_t, having simpler properties, by a relationship

$$\delta(B)\hat{g}_t = \Omega(B)a_t. \tag{B.18}$$

The stochastic models previously considered are of this kind, with a_t a source of white noise. Since (B.18) may be written:

$$\hat{g}_t = Q^{-1}(B)\theta(B)a_t ,$$

it is assumed that g_t could be generated by passing white noise through a linear filter with a transfer function $Q^{-1}(B)\theta(B)$.

In summary:

1. A dynamic relationship connecting an output Y and input X in terms of a linear filter is given by:

$$Y_t = \gamma_0 X_t + \gamma_1 X_{t-1} + \gamma_2 X_{t-2} + \ldots$$

$$= \gamma(B)X_t$$

where $\gamma(B)$ is the transfer function of the filter.

2. In turn, $\gamma(B)$ can frequently be represented with brevity and with sufficient accuracy by a ratio of two polynomials of low degree in B: $\gamma(B)=\delta^{-1}(B)\Omega(B)$ so that the dynamic input-output equation may be written

$$\delta(B)Y_t = \Omega(B)X_t.$$

3. It is postulated that a series g_t, in which successive values are highly dependent can be represented by passing white noise a_t through such a dynamic system in which certain of the roots of $\delta(B)=0$ are unity. This notion yields the autoregressive integrated moving average model:

$$Q(B)g_t = \theta(B)a_t.$$

For further mathematical detail about the models, see Box and Jenkins, particularly Chapters 3, 4, and 9.

APPENDIX C

BUILDING FILES ON THE CREATE SYSTEM

APPENDIX C

BUILDING FILES ON THE CREATE SYSTEM

The program contained in this appendix was designed to build files that can be used by the AFIT.LIB Time Series Analysis programs. Data may be entered into the program directly from the terminal or may be generated from random number generators which are built into the program. The file may be created in either the format that is required for time series analysis model identification (AFIT.LIB/UNIDEN,R) or for time series analysis model parameter estimation and forecasting (AFIT.LIB/UNEST,R). Detailed instructions for using the program are contained in Appendix E.

```
10*RUN *=(ULIB)GRADLIB/TSS,R
20 CHARACTER FMT*80,FILENAME*12,ANS*3
30 PRINT,"WELCOME TO THE WONDERFUL WORLD OF TIME SERIES ANALYSIS½½½½"
40 PRINT,"IS THIS YOUR FIRST ENCOUNTER WITH THE PLEASURES THAT DEFIE"
50 PRINT,"THE IMAGINATIONS OF MEN?(YES OR NO)"
60 READ,ANS; IF(ANS.EQ."NO")GO TO 99
70 PRINT,"SO YOU ARE ADVENTURING INTO NEW AND UNKNOWN WATERS IN THE SEA"
80 PRINT,"OF KNOWLEDGE THAT CAN BE GAINED IN THIS BRIEF SOJOURN UPON EARTH."
90 PRINT,"BEFORE VENTURING INTO THE POSSIBLE ROUGH SEAS THAT LIE AHEAD, IT "
100 PRINT,"MIGHT BE WISE TO ACQUIRE A LIFE JACKET.  THE BEST ONE AVAILABLE "
110 PRINT,"CAN BE FOUND WITH DAN REYNOLDS OR MAJOR MIKE PEARSON OR THEIR"
120 PRINT,"SUCCESSORS.  THE BRAND NAME ON THIS LIFEJACKET IS "
130 PRINT," "
140 PRINT,"*½%%&&*********  AZTECHNICAL REPORT ************&&%%½*"
150 PRINT,"                        B Y    "
160 PRINT,"*******    G E N E   S C H R O E D E R  *********"
170 PRINT,"                       A N D   "
180 PRINT,"******* B R U C E   C H R I S T E N S E N*******"
190 PRINT," "
200 PRINT,"***  A  GUIDE TO USE OF TIME SERIES ANALYSIS ON CREATE ***"
210 PRINT," "
220 PRINT," OR EVEN BETTER YET, THE MAE WEST LIFE JACKET OF  TIME"
230 PRINT," SERIES ANALYSIS,  DESIGNED TO SAVE LIVES IN THE TURBULENT"
240 PRINT,"WATERS OF KNOWLEDGE, OBTAIN FOR YOUR VERY OWN A COPY OF THEIR"
250 PRINT," "
260PRINT," "
270 PRINT,"  ½½½½½½½½½½½½½½  *F*A*N*T*A*B*U*L*O*U*S*   ½½½½½½½½½½½½½½½½"
280 PRINT," "
290 PRINT,"                    T  H  E  S  I  S  "
300 PRINT," "
310 PRINT," "
320 PRINT,"SLSR# 27-76B. IF NOTHING ELSE IT WILL CONSTITUTE DEAD"
330 PRINT,"WEIGHT THAT CAN BE THROWN OVERBOARD AS THE SHIP SINKS."
340 PRINT,"IF YOU HAVEN'T BEEN DISCOURAGED, DON'T SAY WE DIDN'T TRY."
```

```
350 PRINT,"AS YOU LAUNCH INTO THIS NEW ADVENTURE WE SAY TO YOU"
360 PRINT," "
370 PRINT," "
380 PRINT,"                    *********** GOOD LUCK *************            "
390 PRINT," "
400 PRINT," "
410 PRINT," "
420 PRINT," "
430 PRINT,"                   (YOU ARE  GOING TO NEED IT)     "
440 PRINT," "
450 PRINT," "
460 PRINT,"WE WERE ONLY KIDDING. YOU SHOULD FIND YOUR EXPOSURE TO"
470 PRINT,"THE BOX AND JENKINS TIME SERIES ANALYSIS ON CREATE TO BE"
480 PRINT,"A GRATIFYING EXPERIENCE.  IT WILL EXTEND YOUR HORIZONS OF"
490 PRINT,"AVAILABLE FORECASTING TECHNIQUES CONSIDERABLELY.  IT IS NOT "
500 PRINT,"THE CURE-ALL OF FORECASTING.  WE SINCERELY HOPE THAT THIS"
510 PRINT,"PROGRAM AND THE AFOREMENTIONED WRITINGS OF OURS WILL BE "
520 PRINT,"BENEFICIAL TO YOU IN YOUR EFFORTS TO UNDERSTAND TIME"
530 PRINT,"SERIES ANALYSIS."
540 99 PRINT,"ARE YOU BUILDING FILE FORECASTING?"
550 DIMENSION A(500),PA(25),NT(5),MFAC(3)
560 DIMENSION INC(12),IOPA(25)
570 DIMENSION IND(30),IIOD(30)
580 READ,ANS;IF(ANS.EQ."YES")GO TO 2003
590 PRINT,"TYPE FILE NAME (ENDING WITH A SEMI-COLON)";READ,FILENAME
600 CALL ATTACH(11,FILENAME,3,0,ISTAT,);CALL STATUS(ISTAT,$311)
610 CALL FMEDIA(11,2);GOTO 200
620 311 STOP "ATTACH BOMBED"
630 200 X=RND(12345)
640 PRINT,"HOW MANY OBSERVATIONS?";READ,NOB
650 PRINT,"TYPE IN FORMAT (C.R.=(V))";READ,FMT
660 IF(FMT.EQ." ")FMT="(V)"
670 PRINT,"YOUR FORMAT IS";PRINT,FMT
680 PRINT,"DO YOU WANT INSTRUCTIONS ON FILE BUILDING?"
```

```
690 READ,ANS;IF(ANS.EQ."NO")GO TO 2000
700 PRINT,"WHAT IS DATA TRANSFORMATION PARAMETER(TLAM)"
710 PRINT,"TLAM=1.0000, THE SERIES Z REMAINS AS READ IN,TLAM=0"
720 PRINT,"TRANSFORMED SERIES IS LN(Z+TM). ANY OTHER VALUE OF TLAM"
730 PRINT,"THE TRANSFORMED SERIES IS (Z+TM)**TLAM"
740 READ,TLAM
750 PRINT,"ENTER DATA TRANSFORMATION PARAMETER(TM).  THIS VALUE"
760 PRINT,"WILL BE ADDED TO EACH VALUE OF THE SERIES Z BEFORE TRANS-"
770 PRINT,"FORMATION IS MADE.  ENTER .0000 IF NONE."
780 READ,TM
790 PRINT,"ENTER NUMBER OF DIFFERENCE FACTORS(NDIFAC) OR TYPES"
800 PRINT,"(INTEGER VALUES ONLY)."
810 READ,NDIFAC
820 PRINT,"ENTER ARRAY CONTAINING THE NDIFACS NUMBERS OF DIFFERENCES"
830 PRINT,"THIS ND NUMBER, GENERALLY 1.  IT MUST NEVER BE SMALLER"
840 PRINT,"THAN NDIFAC.AUTOCORRELATION FUNCTION WILL BE CALCULATED"
850 PRINT,"FOR ND(1)+1 SERIES, ND(2),ND(3),ETC. ARE USED TO FORM"
860 PRINT,"A NEW ORIGINAL SERIES FROM Z."
870 READ,(IND(J),J=1,NDIFAC)
880 PRINT,"ENTER ARRAY CONTAINING NDIFAC ORDERS OF DIFFERENCES"
890 PRINT,"OF EACH TYPE DESIRED(IOD),I.E. THE VALUES OF S IN"
900 PRINT,"(1-B**S); NORMALLY THE VALUE IS 1, AGAIN AN INTEGER"
910 PRINT,"VALUE IS DESIRED."
920 READ,(IIOD(J),J=1,NDIFAC)
930 PRINT,"ENTER THE NUMBER OF AUTOCORRELATIONS DESIRED (NAC)"
940 READ,NAC
950 PRINT,"ENTER THE NUMBER OF PARTIAL AUTOCORRELATIONS(NPAC) DESIRED"
960 PRINT," THIS CANNOT EXCEED NAC."
970 READ,NPAC
980 PRINT,"ENTER THE NUMBER OF AUTOCORRELATIONS PRINTED PER LINE (NAPL)"
990 PRINT,"BETWEEN 1 AND 12 INCLUSIVE."
1000 READ,NAPL
1010 PRINT,"ENTER THE NUMBER OF AUTOCORRELATIONS TO BE USED IN CALCULATING"
1020 PRINT,"THE CHI-SQUARE STASTIC (NCHI).  SET LESS THAN EQUAL 0 IF NOT"
```

```
1030 PRINT,"WANTED, MAXIMUM VALUE IS NAC."
1040 READ,NCHI
1050 PRINT,"ENTER VALUE OTHER THAN 0,PREFERABLY 1, IF YOU WISH THE"
1060 PRINT,"STANDARD ERRORS OF AUTOCORRELATIONS TO BE CALCULATED."
1070 READ,MCSE
1080 PRINT,"ENTER VALUE FOR ILDID, SET=0 TO SUPPRESS  LISTING OF DATA."
1090 READ,ILDID
1100 PRINT,"ENTER VALUE FOR IPDID, SET=0 TO SUPPRESS PLOTTING OF DATA."
1110 READ,IPDID
1120 PRINT,"ENTER VALUE FOR MPRINT,SET=0 TO SUPPRESS OUTPUT OF ALL STATISTICS."
1130 READ,MPRINT
1140 PRINT,"ENTER VALUE FOR IWTPA,SET=0 TO SUPPRESS PLOTTING OF "
1150 PRINT,"AUTOCORRELATIONS."
1160 READ,IWTPA
1170 GO TO 2001
1180 2000 PRINT,"PLEASE ENTER VALUES FOR TLAM, TM, NDIFAC"
1190 PRINT,"ND,IOD"
1200 PRINT,"NAC,NPAC,NAPL,NCHI"
1210 PRINT,"MCSE,ILDID,IPDID,MPRINT,IWTPA."
1220 READ,TLAM,TM,NDIFAC;READ,((IND(J),IIOD(J)),J=1,NDIFAC)
1230 READ,NAC,NPAC,NAPL,NCHI
1240 READ,MCSE,ILDID,IPDID,MPRINT,IWTPA
1250  2001 PRINT,"ARE YOU ENTERING DATA FROM THE TERMINAL?"
1260 READ,ANS
1270 IF(ANS.EQ."NO") GO TO 4444
1280 PRINT,"ENTER DATA POINTS "
1290 READ,(A(I),I=1,NOB); GO TO 190
1300 4444 PRINT,"FOR RANDOM GENERATOR, ENTER 1 FOR POISSON"
1310 PRINT,"2 FOR INCREASING LINEAR POISSON, 3 FOR "
1320 PRINT,"DECREASING LINEAR POISSON, 4 FOR RANDOM"
1330 PRINT,"LINEAR , 5 FOR SINE, 6 FOR EXPONENTIAL, "
1340 PRINT,"7 FOR HYPERGEOMETRIC DISTRIBUTIONS OF REPARABLE"
1350 PRINT,"GENERATIONS."
1360 PRINT," "
```

```
READ,IRG
GO TO(1,3,5,90,11,13,19),IRG
1 DO 2 I=1,NOB
A(I)=POISSON(10.,1.)
2 CONTINUE
GO TO 190
3 DO 6 I=1,NOB
A(I)=POISSON(10+I/12.,1.)
6 CONTINUE
GO TO 190
5 DO 7 I=1,NOB
A(I)=POISSON(40-I/6.,1.)
7 CONTINUE
GO TO 190
90 DO 8 I=1,12
A(I)=POISSON(20+I/3.,1.)
8 CONTINUE
DO 101 I=13,18
A(I)=POISSON(24-(I-12)/3.,1.)
101 CONTINUE
DO 102 I=19,30
A(I)=POISSON(22+(I-18)/3.,1.)
102 CONTINUE
DO 103 I=31,36
A(I)=POISSON(26-(I-30)/3.,1.)
103 CONTINUE
DO 104 I=37,48
A(I)=POISSON(24+(I-36)/3.,1.)
104 CONTINUE
DO 105 I=49,54
A(I)=POISSON(28-(I-48)/3.,1.)
105 CONTINUE
DO 106 I=55,66
A(I)=POISSON(26+(I-54)/3.,1.)
```

```
1710 106 CONTINUE
1720 DO 107 I=67,72
1730 A(I)=POISSON(30-(I-66)/3.,1.)
1740 107 CONTINUE
1750 DO 108 I=73,84
1760 A(I)=POISSON(28+(I-72)/3.,1.)
1770 108 CONTINUE
1780 DO 109 I=85,90
1790 A(I)=POISSON(32-(I-84)/3.,1.)
1800 109 CONTINUE
1810 DO 111 I=91,102
1820 A(I)=POISSON(30+(I-90)/3.,1.)
1830 111 CONTINUE
1840 DO 112 I=103,108
1850 A(I)=POISSON(34-(I-102)/3.,1.)
1860 112 CONTINUE
1870 DO 113 I=109,NOB
1880 A(I)=POISSON(32+(I-108)/3.,1.)
1890 113 CONTINUE
1900 GO TO 190
1910 11 DO 9 I=1,NOB
1920 A(I)=20*SIN(POISSON(10.,1.))+50
1930 9 CONTINUE
1940 GO TO 190
1950 13 DO 17 I=1,NOB
1960 A(I)=EXPONT(5.)
1970 17 CONTINUE
1980 GO TO 190
1990 19 DO 21 I=1,NOB
2000 A(I)=HYPERG(5,.10,100.)
2010 21 CONTINUE
2020 190 PRINT,"ARE YOU BUILDING FORECAST FILE?"
2030 READ,ANS
2040 IF(ANS.EQ."YES")GO TO 2005
```

```
2050 WRITE(11,301)NOB;WRITE(11,302)FMT;WRITE(11,FMT)(A(I),I=1,NOB)
2060 WRITE(11,303)FILENAME;WRITE(11,307)TLAM,TM,NDIFAC
2070 WRITE(11,304)((IND(J),IIOD(J)),J=1,NDIFAC)
2080 WRITE(11,304)NAC,NPAC,NAPL,NCHI
2090 WRITE(11,304)MCSE,ILDID,IPDID,MPRINT,IWTPA
2100 301 FORMAT(I5);302 FORMAT(A80)
2110 303 FORMAT(A8,1X,"DISTRIBUTION  10 YEARS DATA")
2120 304 FORMAT(16I5)
2130 307 FORMAT(2F8.4,I5)
2140 REWIND 11
2150 GO TO 5000
2160 2003 PRINT,"ENTER FILENAME ENDING WITH SEMI-COLON."
2170 READ,FILENAME
2180 CALL ATTACH(12,FILENAME,3,0,ISTAT,);CALL STATUS(ISTAT,$312)
2190 CALL FMEDIA(12,2);GO TO 201
2200 312 STOP "ATTACH BOMBED, NO FILE THERE"
2210 201 X=RND(12345)
2220 DIMENSION ND(30),IOD(30),ZN(200)
2230 PRINT,"ENTER THE NUMBER OF DATA POINTS TO"
2240 PRINT,"BE USED AS THE DATA BASE."
2250 READ,NOB
2260 PRINT,"TYPE IN FORMAT (C.R.=(V))";READ,FMT
2270 IF(FMT.EQ." ")FMT="(V)"
2280 PRINT,"YOUR FORMAT IS";PRINT,FMT
2290  PRINT,"DO YOU NEED ASSISTANCE IN BUILDING THIS"
2300 PRINT,"FILE FOR FORECASTING?"
2310 READ,ANS
2320 IF (ANS.EQ."NO")GO TO 2004
2330 PRINT,"ENTER VALUE FOR TLAM,THIS THE DATA TRANS-"
2340 PRINT,"FORMATION PARAMETER. IF TLAM=1, THE ORIGINAL"
2350 PRINT,"DATA WILL BE USED AS IS, IF TLAM=0,TRANSFORMED "
2360 PRINT,"SERIES IS LN(Z+TM). ANY OTHER VALUES FOR TLAM"
2370 PRINT," WILL BE TRANSFORMED AS FOLLOWS: (Z+TM)**TLAM."
2380 READ,TLAM
```

```
2390 PRINT,"ENTER VALUE FOR TM. THIS IS DATA TRANSFORMATION"
2400 PRINT,"THAT WILL BE ADDED TO ORINAL SERIES Z BEFORE ANY"
2410 PRINT,"TRANSFORMATION IS MADE."
2420 READ,TM
2430 PRINT,"ENTER MFAC(1),MFAC(2),MFAC(3).  MFAC(1), IS "
2440 PRINT,"THE NUMBER OF AUTOREGRESSIVE FACTORS,MFAC(2)"
2450 PRINT,"IS THE NUMBER OF DIFFERENCE FACTORS,MFAC(3) IS"
2460 PRINT,"THE NUMBER OF MOVING AVERAGE FACTORS IN THE TIME"
2470 PRINT,"SERIES."
2480 READ,(MFAC(J),J=1,3)
2490 PRINT,"ENTER VALUE FOR ND. ND IS GENERALLY 1, IT IS THE"
2500 PRINT,"ARRAY CONTAINING THE MFAC(2) NUMBER OF DIFFERENCES."
2510 PRINT,"MINIMUM SIZE=MFAC(2)."
2520 MAX=MFAC(2)
2530 IF(MAX.EQ.0) GO TO 1440
2540 READ,(ND(J),J=1,MAX)
2550 PRINT,"MFAC(2)  ORDERS OF DIFFERENCE.  IT IS NORMALLY 1,"
2560 PRINT,"MINIMUM SIZE IS MFAC(2)."
2570 READ,(IOD(J),J=1,MAX)
2580 NP=0;
2590 1440 PRINT,"ENTER VALUES FOR INC.  INC IS THE ARRAY CONTAINING"
2600 PRINT,"(MFAC(1)+MFAC(3)+2) NUMBERS OF EACH OF THE "
2610 PRINT,"SPECIFIED TYPES OF PARAMETERS IN THE MODEL TO BE USED."
2620 PRINT,"MINIMUM NUMBER OR SIZE IS (MFAC(1)+MFAC(3)+2)."
2630 READ,
2640 DO 1500 J=1,MAX
2650 NP=NP+INC(J)
2660 1500 CONTINUE
2670 IF(NP.EQ.0) GO TO 1441
2680 PRINT,"ENTER VALUES FOR IOPA.IOPA IS THE ARRAY CONTAINING"
2690 PRINT,"THE ORDER OF EACH PARAMETER FROM LEFT TO RIGHT IN"
2700 PRINT,"THE TIME SERIES MODEL.  NUMBER OF VALUES WILL BE"
2710 PRINT,"THE TOTAL OF THE SUM OF THE INC."
2720 READ,(IOPA(J),J=1,NP)
```

```
2730 PRINT,"ENTER VALUE FOR PA. PA IS THE ARRAY CONTAINING"
2740 PRINT,"THE INITIAL ESTIMATES OF THE PARAMETERS FOR FORECASTING"
2750 PRINT,"CONTAINS PREVIOUSLY ESTIMATED PARAMETER VALUES."
2760 PRINT,"MINIMUM SIZE IS THE SUM OF INC."
2770 READ,(PA(J),J=1,NP)
2780 1441 PRINT,"ENTER VALUES FOR EPS1. EPS1 IS THE MAXIMUM"
2790 PRINT,"CHANGE IN RELATIVE SUM OD SQUARES ALLOWED BEFORE"
2800 PRINT,"ITERATION STOPS. SET=.00 TO SUPPRESS."
2810 READ,EPS1
2820 PRINT,"ENTER VALUE FOR EPS2. EPS2 IS MAXIMUM RELATIVE "
2830 PRINT,"CHANGE IN EACH PARAMETER BEFORE ITERATION STOPS."
2840 PRINT,"SET=.00 TO SUPPRESS."
2850 READ,EPS2
2860 PRINT,"ENTER VALUE FOR MIT. MIT IS THE MAXIMUM NUMBER"
2870 PRINT,"OF ITERATIONS ALLOWED FOR ESTIMATION. NOT TO "
2880 PRINT,"EXCEED 999."
2890 READ,MIT
2900 PRINT,"ENTER VALUE FOR ILDEST. ILDEST IS THE LISTING "
2910 PRINT,"OF DATA BY ESTIMATION ROUTINE. SET=0 TO SUPPRESS."
2920 READ,ILDEST
2930 PRINT,"ENTER VALUE FOR IPDEST. IPDEST IS THE PLOTTING OF"
2940 PRINT,"OF DATA BY ESTIMATION. SET=0 TO SUPPRESS."
2950 READ,IPDEST
2960 PRINT,"ENTER VALUES FOR IPRES. IPRES IS PLOTTING OF "
2970 PRINT,"RESIDUALS.SET=0 TO SUPPRESS."
2980 READ,IPRES
2990 PRINT,"ENTER VALUE FOR NAC. NAC IS THE NUMBER OF"
3000 PRINT,"AUTOCORRELATIONS TO BE RUN."
3010 READ,NAC
3020 PRINT,"ENTER VALUE FOR NPAC. NPAC IS THE NUMBER OF "
3030 PRINT,"PARTIAL AUTO CORRELATIOS TO BE RUN. CANNOT"
3040 PRINT,"EXCEED NAC."
3050 READ,NPAC
3060 PRINT,"ENTER VALUE FOR MCSE. SET=0 IF DO NOT WANT"
```

```
3070 PRINT,"STANDARD ERRORS OF AUTOCORRELATIONS CALCULATED."
3080 READ,MCSE
3090 PRINT,"ENTER VALUE FOR NAPL. NAPL IS THE NUMBER OF "
3100 PRINT,"AUTOCORRELATIONS TO BE PRINTED PER LINE."
3110 PRINT,"BETWEEN 1 AND 12."
3120 READ,NAPL
3130 PRINT,"ENTER VALUE FOR IWTPA. SET=0 TO SUPPRESS PLOTTING"
3140 PRINT,"OF RESIDUAL AUTOCORRELATIONS."
3150 READ,IWTPA
3160 PRINT,"ENTER VALUE FOR NCHI. NCHI IS THE NUMBER OF "
3170 PRINT,"AUTOCORRELATIONS TO BE USED IN CALCULATING A"
3180 PRINT,"CHI-SQUARE STASTIC. SET=0 TO SUPPRESS."
3190 READ,NCHI
3200 PRINT,"ENTER VALUE FOR NF. NF IS THE NUMBER OF FORECASTS"
3210 PRINT,"DESIRED.  BETWEEN 0 AND 300 INCLUSIVE."
3220 READ,NF
3230 PRINT,"ENTER THE NUMBER OF TIME ORIGINS FOR THE FORECAST(NTO)"
3240 READ,NTO
3250 PRINT,"ENTER THE NUMBER OF NEW OBSERVATIONS (NU) TO BE"
3260 PRINT,"READ INTO THE PROGRAM FROM UPDATES OF THE FORECAST."
3270 PRINT,"MAXIMUM NUMBER IS NF-1."
3280 READ,NU
3290 PRINT,"ENTER VALUE FOR ICI. ICI IS CONTROL ON CONFIDENCE"
3300 PRINT,"INTERVAL WIDTH. 1,2,3,4,5 FOR 50,75.90,95,99 % LIMITS"
3310 READ,ICI
3320 PRINT,"ENTER VALUE FOR ILDFCA. SET=0 TO SUPPRESS LISTING"
3330 PRINT,"OF DATA BY FORECASTING ROUTINE."
3340 READ,ILDFCA
3350 PRINT,"ENTER VALUE FOR IPDFCA.SET=0 TO SUPPRESS PLOTTING."
3360 READ,IPDFCA
3370 PRINT,"ENTER VALUE FOR IWTPF. SET=0 TO SUPPRESS PLOTTING "
3380 PRINT,"OF FORECASTS."
3390 PRINT,"ENTER VALUES FOR NT. NT IS THE ARRAY  OF "
3400 PRINT,"MINIMUM SIZE NTO, CONTAINING FORECAST TIME"
```

```
3410 PRINT,"ORIGINS, EAC ORIGIN MUST BE AT LEAST THE BIGGEST"
3420 PRINT,"BACKORDER IN THE EQUATION. MAXIMUM VALUE IS 200."
3430 PRINT," AND LESS THAN NOB."
3440 READ,NT
3450 GO TO 2001
3460 2004 PRINT,"ENTER VALUES FOR TLAM,TM,MFAC(1),MFAC(2),MFAC(3)"
3470 READ,TLAM,TM,(MFAC(J),J=1,3)
3480 MAX=MFAC(2)
3490 IF (MAX.EQ.0) GO TO 800
3500 PRINT,"ENTER VALUES FOR ND, IOD."
3510 READ,((ND(J),IOD(J)),J=1,MAX)
3520 800 PRINT,"ENTER VALUES FOR INC. MUST = SUM OF MFAC(1)+MFAC(3) +2"
3530 MAX=MFAC(1)+MFAC(3)+2
3540 READ,(INC(J),J=1,MAX)
3550 NP=0
3560 DO 1502 J=1,MAX
3570 NP=NP+INC(J)
3580 1502 CONTINUE
3590 IF(NP.EQ.0) GO TO 1442
3600 PRINT,"ENTER VALUES FOR IOPA.(MUST HAVE #=SUM OF INC.)"
3610 READ,(IOPA(J),J=1,NP)
3620 PRINT,"ENTER VALUES FOR PA. SAME SIZE RESTRICTIONS AS"
3630 PRINT,"FOR IOPA."
3640 READ,(PA(J),J=1,NP)
3650 1442 PRINT,"ENTER VALUES FOR EPS1,EPS2,MIT,ILDEST,IPDEST,IPRES."
3660 READ,EPS1,EPS2,MIT,ILDEST,IPDEST,IPRES
3670 PRINT,"ENTER VALUES FOR NAC,NPAC,MCSE,NAPL,IWTPA,NCHI."
3680 READ,NAC,NPAC,MCSE,NAPL,IWTPA,NCHI
3690 PRINT,"ENTER VALUES FOR NF,NTO,NU,ICI,ILDFCA,IPDFCA,IWTPF."
3700 READ,NF,NTO,NU,ICI,ILDFCA,IPDFCA,IWTPF
3710 PRINT,"ENTER VALUES FOR NT, # NOT LESS THAN MBO NOR MORE THAN NOB."
3720 READ,(NT(J),J=1,NTO)
3730 IF(NU.EQ.0) GO TO 2001
3740 PRINT,"ENTER DATA TO UPDATE FORECAST. IF YOU"
```

```
3750 PRINT,"DESIRE TO HAVE A RANDOM NUMBER GENERATOR"
3760 PRINT,"FILE THE VALUES FOR YOU, ENTER 0."
3770 READ,ANS; IF(ANS.EQ."0") GO TO 2221
3780 READ,(ZN(I),I=1,NU)
3790 GO TO 2001
3800 2005 WRITE(12,400)NOB
3810 WRITE(12,406)FMT;WRITE(12,FMT)(A(I),I=1,NOB)
3820 WRITE(12,407)FILENAME
3830 WRITE(12,401)TLAM,TM,(MFAC(J),J=1,3)
3840 MAX=MFAC(2)
3850 IF(MAX.NE.0) WRITE(12,400)((ND(J),IOD(J)),J=1,MAX)
3860 MAX=MFAC(1)+MFAC(3)+2
3870 WRITE(12,400)(INC(J),J=1,MAX)
3880 NP=0
3890 DO 1504 J=1,MAX
3900 NP=NP+INC(J)
3910 1504 CONTINUE
3920 IF(NP.NE.0) WRITE(12,400)(IOPA(J),J=1,NP)
3930 IF(NP.NE.0) WRITE(12,405)(PA(J),J=1,NP)
3940 WRITE(12,401)EPS1,EPS2,MIT,ILDEST,IPDEST,IPRES
3950 WRITE(12,400)NAC,NPAC,MCSE,NAPL,IWTPA,NCHI
3960 WRITE(12,400)NF,NTO,NU,ICI,ILDFCA,IPDFCA,IWTPF
3970 WRITE(12,400)(NT(I),I=1,NTO)
3980 IF(NU.NE.0) WRITE(12,FMT)(ZN(J),J=1,NU)
3990 401 FORMAT(2F8.4,4I5)
4000 400 FORMAT(16I5)
4010 405 FORMAT(10F8.4)
4020 406 FORMAT(A80)
4030 407 FORMAT(A8,1X,"DISTRIBUTION")
4040 REWIND 12
4050 GO TO 5000
4060 2221 PRINT,"FOR RANDOM GENERATION OF NEW OBSERVATIONS, ENTER 1 FOR POISSON"
4070 PRINT,"2 FOR INCREASING LINEAR POISSON, 3 FOR "
4080 PRINT,"DECREASING LINEAR POISSON, 4 FOR RANDOM"
```

```
4090 PRINT,"LINEAR , 5 FOR SINE, 6 FOR EXPONENTIAL, "
4100 PRINT,"7 FOR HYPERGEOMETRIC DISTRIBUTIONS OF REPARABLE"
4110 PRINT,"GENERATIONS."
4120 PRINT," "
4130 READ,IRG
4140 GO TO(100,300,500,920,1100,1300,1900),IRG
4150 100 DO 42 I=1,NOB
4160 ZN(I)=POISSON(10.,1.)
4170 42 CONTINUE
4180 GO TO 2001
4190 300 DO 46 I=1,NU
4200 ZN(I)=POISSON(10+I/12.,1.)
4210 46 CONTINUE
4220 GO TO 2001
4230 500 DO 47 I=1,NU
4240 ZN(I)=POISSON(40-I/6.,1.)
4250 47 CONTINUE
4260 GO TO 2001
4270 920 DO 48 I=1,12
4280 ZN(I)=POISSON(20+I/3.,1.)
4290 48 CONTINUE
4300 DO 221 I=13,18
4310 ZN(I)=POISSON(24-(I-12)/3.,1.)
4320 221 CONTINUE
4330 DO 222 I=19,30
4340 ZN(I)=POISSON(22+(I-18)/3.,1.)
4350 222 CONTINUE
4360 DO 223 I=31,36
4370 ZN(I)=POISSON(26-(I-30)/3.,1.)
4380 223 CONTINUE
4390 DO 224 I=37,48
4400 ZN(I)=POISSON(24+(I-36)/3.,1.)
4410 224 CONTINUE
4420 DO 225 I=49,54
```

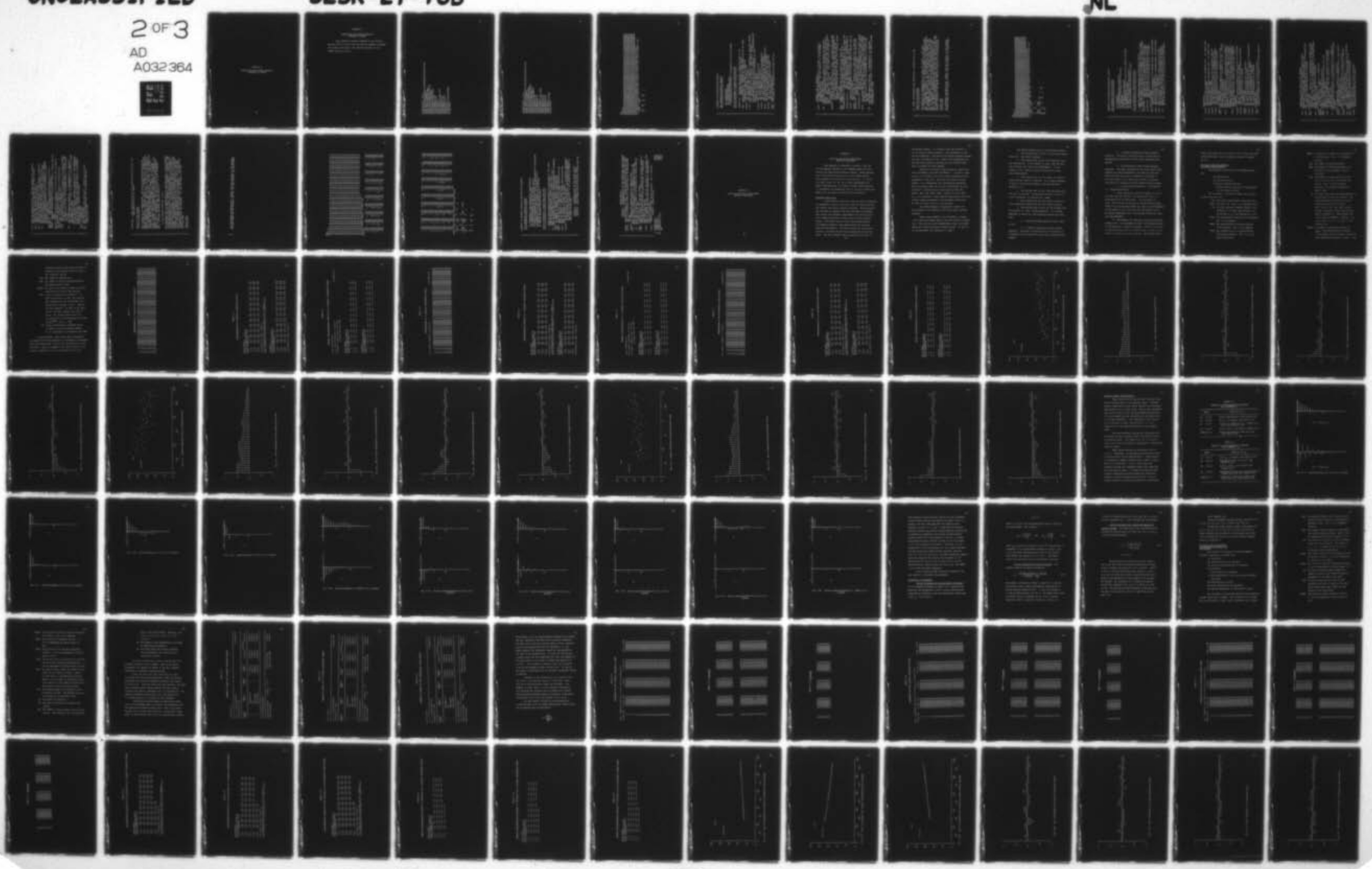
```
4430 ZN(I)=POISSON(28-(I-48)/3.,1.)
4440 225 CONTINUE
4450 DO 226 I=55,66
4460 ZN(I)=POISSON(26+(I-54)/3.,1.)
4470 226 CONTINUE
4480 DO 227 I=67,72
4490 ZN(I)=POISSON(30-(I-66)/3.,1.)
4500 227 CONTINUE
4510 DO 228 I=73,84
4520 ZN(I)=POISSON(28+(i-72)/3.,1.)
4530 228 CONTINUE
4540 DO 229 I=85,90
4550 ZN(I)=POISSON(32-(I-84)/3.,1.)
4560 229 CONTINUE
4570 DO 211 I=91,102
4580 ZN(I)=POISSON(30+(I-90)/3.,1.)
4590 211 CONTINUE
4600 DO 212 I=103,108
4610 ZN(I)=POISSON(34-(I-102)/3.,1.)
4620 212 CONTINUE
4630 DO 213 I=109,NU
4640 ZN(I)=POISSON(32+(I-108)/3.,1.)
4650 213 CONTINUE
4660 GO TO 2001
4670 1100 DO 49 I=1,NU
4680 ZN(I)=20*SIN(POISSON(10.,1.))+50
4690 49 CONTINUE
4700 GO TO 2001
4710 1300 DO 170 I=1,NU
4720 ZN(I)=EXPONT(5.)
4730 170 CONTINUE
4740 GO TO 2001
4750 1900 DO 41 I=1,NU
4760 ZN(I)=HYPERG(5,.10,100.)
```

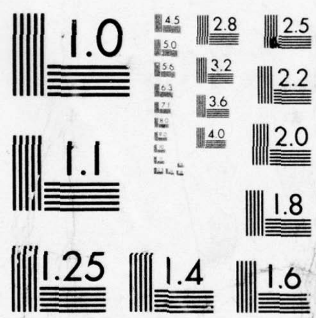
```
41 CONTINUE
GO TO 2001
5000 PRINT,"ARE YOU BUILDING ANOTHER FILE?"
PRINT,"****WARNING******IF YOU HAVE BUILT PREVIOUS FILES"
PRINT,"IN A NON-CONTINOUS (THAT IS BY GOING BACK TO RUN"
PRINT,"STATEMENT)BASIS USING THE RANDOM NUMBER GENERATOR"
PRINT,"IF YOU ATTEMPT TO REPLICATE THE DATA YOU MUST"
PRINT,"RETURN TO RUN LEVEL AGAIN.**************************"
READ,ANS; IF (ANS.EQ."YES") GO TO 99
STOP "OUTPUT COMPLETED";END
```

AD-A032 364 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OHIO SCHO--ETC F/G 15/5
A COMPARATIVE ANALYSIS OF THE D041 SYSTEM AND TIME SERIES ANALY--ETC(U)
SEP 76 B R CHRISTENSEN, G J SCHROEDER

UNCLASSIFIED SLSR-27-76B NL

2 OF 3
AD
A032364





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

APPENDIX D

RUNNING THE TIME SERIES ANALYSIS PROGRAMS ON CREATE

APPENDIX D

RUNNING THE TIME SERIES ANALYSIS PROGRAMS ON CREATE

This appendix contains examples of the control program used to run the Box and Jenkins computer programs, the example data decks, and comment programs on the CREATE computer system.

```
0010##M,R(SL) ;,8,16
0020$;IDENT;WP1190,AFITSL,CHRISTENSEN/SCHROEDER,76B
0030$;USERID;76A85$HW11
0040$;OPTION;FORTRAN,NOMAP
0050$;FORTY;NFORM,NLNO,NLSTIN
0060$;SELECTA;76A85/SCH1
0070$;SELECTA;TIME/SUBS,R
0071$;REMOTE;P*,XA
0072$;REMOTE;$$,SL
0080$;EXECUTE
0082$;LIMITS;10,39K,,5000
0083$;REMOTE;P*,XA
0084$;REMOTE;52,SL
0085$;REMOTE;42,SL
0090$;DATA;I*
0100$;SELECT;76A85/XLOGNORM
0110$;ENDJOB
```

```
0010##M,R(SL) ;,8,16
0020$;IDENT;WP1190,AFITSL,SCHROEDER/CHRISTENSEN,76B
0030$;USERID;76A85$HW11
0040$;OPTION;FORTRAN,NOMAP
0050$;FORTY;NFORM,NLNO,NLSTIN
0060$;SELECTA;TIME/UNEST,R
0070$;SELECTA;TIME/SUBS,R
0071$;REMOTE;P*,XX
0072$;REMOTE;$$,SL
0080$;EXECUTE
0082$;LIMITS;10,39K,,9950
0083$;REMOTE;P*,XX
0084$;REMOTE;52,SL
0085$;REMOTE;42,SL
0090$;DATA;I*
0100$;SELECTA;DATAEX2
0110$;ENDJOB
```

```
157
(20F4.3)
24562372231026132650252723342606285029613000323032103165314031133042331631653404
35783591333731022598156213541126104608810962168624842793275628142837271228522960
28513247324333583998411742094572443639543439324433922641239622862489242623842272
23022408242023272288235922682402230423502458261727462752271927352694271929452837
279227512803285629142916289729092920299531433332033793453352235233529353235533484
34823478347935063527357536243856382839293942393238953810383138363912403240824362
45964670462646114642453948554932535653875344500747594554428838523640348043084275
445145884762501250814969514453655621554453825095520253345492591661 77
U S TREASURY BILLS INTEREST RATE, MONTHLY JANUARY 1956 THROUGH JANUARY 1969
  1.0000   .0000    1
    1     1
   36    36    12    36
    1     1     1     1     1
```

```
C     COMMENTS ON IDENTIFICATION EXAMPLE
C
C     HOW DIMENSIONS RESTRICT PROBLEM
C     *******************************
C
C        500 OBSERVATIONS
C        150 AUTOCORRELATIONS
C        150 PARTIAL AUTOCORRELATIONS
C        2 AUTOCORRELATION FUNCTIONS OF DIFFERENCES OF ORIGINAL SERIES
C
C     LIST OF VARIABLES (* MEANS USER SUPPLIED)
C     *****************************************
C
C     E(ND(1)+1) - CONTAINS MEAN OF ORIG. SERIES AND EACH OF THE
C                  REQUESTED DIFFERENCES OF ORDER IOD(1)
C    *FMZ - ARRAY OF SIZE 20.  CONTAINS THE FORMAT SPECIFICATION FOR INPUT
C           TO PROGRAM OF TIME SERIES TO BE IDENTIFIED
C    *ILDID - SET = 0 TO SUPPRESS LISTING OF DATA
C             ANY OTHER VALUE DATA IS LISTED
C    *IOD - ARRAY CONTAINING NDIFAC ORDERS OF DIFFERENCES OF EACH TYPE DESIRED,
C           I.E., THE VALUES OF S IN (1-B**S).  ORIG. SERIES BECOMES
C           Z*(1-B**IOD(2))**ND(2)*(1-B**IOD(3))**ND(3)*....  AN AUTOCORRELATION
C           FUNCTION IS CALCULATED FOR ND(1) DIFFERENCES OF TYPE
C           (1-B**IOD(1)) OF THIS SERIES
C    *IPDID - SET = 0 TO SUPPRESS PLOTTING OF DATA
C             ANY OTHER VALUE DATA IS PLOTTED
C    *IWTPA - SET = 0 TO SUPRESS PLOTTING OF AUTOCORRELATIONS
C             ANY OTHER VALUE AUTOCORRELATIONS ARE PLOTTED
C    *MCSE - SET=0 DO NOT WISH STANDARD ERRORS OF AUTOCORRELATIONS
C            ANY OTHER VALUE STANDARD ERRORS ARE CALCULATED
C    *MPRINT - SET = 0 TO SUPPRESS OUTPUT OF ALL STATISTICS
C              ANY OTHER VALUE ALL STATISTICS OUTPUT
C    *NAC - NUMBER OF AUTOCORRELATIONS
C    *NAPL - THE NUMBER OF AUTOCORRELATIONS PRINTED PER LINE
```

```
C           BETWEEN 1 AND 12 INCLUSIVE
C     *NCHI - THE NUMBER OF AUTOCORRELATIONS TO BE USED IN CALCULATING A
C           CHI-SQUARE STATISTIC.  SET LESS THAN OR = 0 IF NOT WANTED.
C           MAXIMUM VALUE IS NAC.
C     *ND - ARRAY CONTAINING THE NDIFAC NUMBERS OF DIFFERENCES OF EACH TYPE
CZ          DESIRED.  AUTOCORRELATION FUNCTION WILL BE CALCULATED FOR ND(1)+1
C           SERIES.  ND(2), ND(3), ETC., ARE USED TO FORM A NEW 'ORIGINAL SERIES'
C           FROM Z.  MINIMUM SIZE IS NDIFAC.  SOME GRAPH TITLES NOT GOOD IF
C           ND(1) EXCEEDS 5.  (SEE IOD)
C     *NDIFAC - NUMBER OF DIFFERENCE FACTORS OR TYPES.  AUTOCORRELATION FUNCTION
C               IS CALCULATED FOR ORIG. SERIES AND EACH REQUESTED DIFFERENCE OF
C               TYPE 1.  DIFFERENCE FACTORS BEYOND THE FIRST ARE USED TO
C               DIFFERENCE THE ORIGINAL SERIES TO OBTAIN A NEW 'ORIGINAL SERIES'.
C               (SEE ND, IOD)
C     *NOB - NUMBER OF OBSERVATIONS IN TIME SERIES TO BE IDENTIFIED
C     *NPAC - NUMBER OF PARTIAL AUTOCORRELATIONS. MAXIMUM VALUE IS NAC
C      PHI(NPAC,ND(1)+1) - CONTAINS PARTIAL AUTOCORRELATIONS OF ORIG. SERIES AND
C                          EACH OF THE REQUESTED DIFFERENCES OF ORDER IOD(1)
C      RHO(NAC,ND(1)+1) - CONTAINS AUTOCORRELATIONS OF ORIG. SERIES AND EACH OF
C                         THE REQUESTED DIFFERENCES OF ORDER IOD(1)
C      SCRATC - TEMPORARY STORAGE ARRAY WITH REQUIRED DIMENSION GREATER THAN OR =
C               MAX OF (NOB,2*NPAC)
C     *SERIES - ARRAY OF SIZE 20. CONTAINS TITLE DESCRIBING DATA OR ANALYSIS.
C      SM(ND(1)+1) - CONTAINS ST. DEVIATIONS OF ORIG. SERIES AND EACH OF THE
C                    REQUESTED DIFFERENCES OF ORDER IOD(1)
C      STE(NAC,ND(1)+1) - CONTAINS ST. ERRORS OF AUTOCORRELATIONS OF ORIG. SERIES
C                         AND EACH OF THE REQUESTED DIFFERENCES OF ORDER IOD(1)
C     *TLAM - DATA TRANSFORMATION PARAMETER.  IF TLAM=1.0, THE SERIES Z REMAINS
C           AS READ IN.  IF TLAM=0, THE TRANSFORMED SERIES IS LN(Z+TM).  FOR
C           ANY OTHER VALUE OF TLAM, THE TRANSFORMED SERIES IS (Z+TM)**TLAM.
C           (SEE TM)
C     *TM - DATA TRANSFORMATION PARAMETER.  IF TRANSFORMATION IS REQUESTED
C           (SEE TLAM), THIS VALUE IS ADDED TO EACH VALUE OF THE SERIES Z BEFORE
C           TRANSFORMATION IS MADE
```

```
C     *Z - ARRAY FOR ORIGINAL TIME SERIES DATA. MINIMUM SIZE IS NOB.
C
C      OUTPUT OF THIS EXAMPLE
C      **********************
C
C      THE SERIES OF 157 OBSERVATIONS IS LISTED AND PLOTTED.  THE TITLE
C      'U S TREASURY BILLS INTEREST RATE, MONTHLY JANUARY 1956 THROUGH JANUARY
C      1969 ' IS ASSIGNED TO THE RUN.  36 AUTOCORRELATIONS, WITH STANDARD ERRORS,
C      ARE CALCULATED, AND PRINTED 12 PER LINE, FOR THE ORIGINAL SERIES Z AND
C      THE DIFFERENCED SERIES (1-B**1)**1*Z. THE 'ORIGINAL SERIES' IS THE DATA
C      FED IN, SINCE TLAM=1.0, INDICATING NO ARITHMETIC TRANSFORMATION OF THE
C      DATA, AND NDIFAC=1, INDICATING NO DIFFERENCING TRANSFORMATIONS OF THE
C      DATA.  36 PARTIAL AUTOCORRELATIONS ARE CALCULATED AND PRINTED 12 PER LINE.
C      ALL CORRELATIONS CALCULATED ARE GRAPHED, AND ALL STATISTICS CALCULATED ARE
C      PRINTED, INCLUDING A CHI-SQUARE STATISTIC BASED ON 36 AUTOCORRELATIONS.
C
C      SPECIAL NOTE
C      ************
C
C      THE STATEMENT  BEGINNING WITH 'COMMON' MUST BE INCLUDED IN THE PROGRAM.
C      VARIABLE NAMES MAY BE CHANGED, BUT NOT THE NAME BETWEEN THE SLASH MARKS.
C
```

```
157
(20F4.3)
24562372231026132650252723342606285029613000323032103165314031133042331631653404
35783591333731022598156213541126104608810962168624842793275628142837271228522960
28513247324333583998411742094572443639543439324433922641239622862489242623842272
23022408242023272288235922682402230423502458261727462752271927352694271929452837
27922751280328562914291628972909292029953143332033793453352235233529353235533484
34823478347935063527357536243856382839293942393238953810383138363912403240824362
45964670462646114642453948554932535653875344500747594554428838523640348043084275
4451458847625012508149695144536556215544538250955202533454925916617 7
U S TREASURY BILLS INTEREST RATE, MONTHLY JANUARY 1956 THROUGH JANUARY 1969
  1.0000    .0000    1    1    0
    1    1
    2    0    0
    1    2
   .8000    .5000
   .0000    .0040  100    1    0    1
   24   24    1    8    1   24
   20    2    0    4    0    0    1
   85  157
```

```
C      COMMENTS OF ESTIMATION-FORECASTING EXAMPLE
C
C      HOW DIMENSIONS RESTRICT PROBLEM
C      ********************************
C
C         25 PARAMETERS
C         150 AUTOCORRELATIONS OF RESIDUALS
C         150 PARTIAL AUTOCORRELATIONS OF RESIDUALS
C         300 FORECASTS
C         5 TIME ORIGINS FOR FORECASTS
C         25 NEW OBSERVATIONS TO USE IN CALCULATING UPDATED FORECASTS
C         500 OBSERVATIONS IN SERIES
C         800 TOTAL OF OBSERVATIONS AND FORECASTS
C         200 MAXIMUM BACKORDER ON EITHER SIDE OF MODEL
C
C      LIST OF VARIABLES (* MEANS USER SUPPLIED)
C      *****************************************
C
C      A - ARRAY OF CALCULATED RESIDUALS. MINIMUM SIZE IS NOB+NF
C      C - ARRAY FOR STORAGE OF COEFFICIENTS OF EXPANDED LEFT SIDE OF MODEL
C      CF - ARRAY FOR STORAGE OF COEFFICIENTS OF EXPANDED RIGHT SIDE OF MODEL
C      CLL(NF,NTO) - ARRAY CONTAINING THE NF LOWER CONFIDENCE LIMITS FOR
C                    FORECASTS AT THE J TH TIME ORIGIN IN ITS J TH COLUMN
C      CUL(NF,NTO) - ARRAY CONTAINING THE NF UPPER CONFIDENCE LIMITS FOR
C                    FORECASTS AT THE J TH TIME ORIGIN IN ITS J TH COLUMN
C      E - MEAN OF RESIDUALS
C     *EPS1 - MAXIMUM CHANGE IN RELATIVE SUM OF SQUARES BEFORE ITERATION STOPS.
C             SET =.00 IF WISH SUPPRESSED
C     *EPS2 - MAXIMUM RELATIVE CHANGE IN EACH PARAMETER BEFORE ITERATION STOPS.
C             SET = .00 IF WISH SUPPRESSED
C     *FMZ - ARRAY OF SIZE 20.  CONTAINS THE FORMAT SPECIFICATION FOR
C            INPUT TO PROGRAM OF TIME SERIES TO BE MODELLED OR FORECAST
C     *ICI - CONTROL ON THE WIDTH OF THE CONFIDENCE LIMITS FOR FORECASTS.
C            ONE OF THE VALUES 1,2,3,4,5 FOR 50,75,90,95,99 PER CENT LIMITS
```

```
C          RESPECTIVELY
C     *IEYON - SET = 0 IF WISH TO SUPRESS THE ESTIMATION STEP.  ANY OTHER VALUE
C              ESTIMATION WILL BE DONE
C     *IFYON - SET = 0 IF WISH TO SUPRESS FORECASTING STEP.  ANY OTHER VALUE
C              FORECASTING WILL BE DONE
C     *ILDEST - SET=0 TO SUPPRESS LISTING OF DATA BY ESTIMATION ROUTINE.
C               ANY OTHER VALUE DATA IS LISTED BY ESTIMATION ROUTINE
C     *ILDFCA - SET=0 TO SUPPRESS LISTING OF DATA BY FORECASTING ROUTINE
C               ANY OTHER VALUE DATA IS LISTED BY FORECASTING ROUTINE
C      ILDID - NO PURPOSE HERE
C     *INC - ARRAY CONTAINING (MFAC(1)+MFAC(3)+2) NUMBERS OF EACH OF THE
C            SPECIFIED TYPES OF PARAMETERS IN THE MODEL TO BE USED.  MINIMUM
C            SIZE IS (MFAC(1)+MFAC(3)+2)
C     *IOD - ARRAY OF MINIMUM SIZE MFAC(2).  CONTAINS THE MFAC(2) ORDERS OF
C            DIFFERENCES OF EACH TYPE OF DIFFERENCE DESIRED IN TIME SERIES
C            MODEL, I.E., THE VALUE OF S IN (1-B**S).  MODEL WILL CONTAIN
C            DIFFERENCE FACTORS (1-B**IOD(1))**ND(1)*(1-B**IOD(2))**ND(2)*....
C     *IOPA - ARRAY OF MINIMUM SIZE NP.  CONTAINS ORDER OF EACH PARAMETER
C             FROM LEFT TO RIGHT IN TIME SERIES MODEL TO BE USED (POWER OF
C             B OPERATOR)
C     *IPDEST - SET = 0 IF WISH TO SUPPRESS PLOTTING OF DATA BY ESTIMATION
C               ROUTINE. ANY OTHER VALUE DATA IS PLOTTED
C     *IPDFCA - SET=0 TO SUPRESS PLOTTING OF DATA BY FORECASTING ROUTINE.
C               ANY OTHER VALUE DATA WILL BE PLOTTED
C      IPDID - NO PURPOSE HERE
C     *IPRES - SET = 0 IF WISH TO SUPRESS PLOT OF RESIDUALS .
C              ANY OTHER VALUE RESIDUALS ARE PLOTTED
C     *IWTPA - SET = 0 IF WISH TO SUPRESS PLOTS OF RESIDUAL AUTOCORRELATIONS.
C              ANY OTHER VALUE AUTOCORRELATIONS ARE PLOTTED
C     *IWTPF - SET = 0 TO SUPRESS PLOTTING OF FORECASTS.
C              ANY OTHER VALUE FORECASTS WILL BE PLOTTED
C      JOD - NO PURPOSE HERE
C      MAX11 - BIGGEST BACKORDER OF LEFT OF MODEL EQUATION + 1
C              ALSO USED AS ERROR SIGNAL, WITH MAX11=-1 IF PROGRAM TAKES AN
```

```
C             ERROR EXIT AT ANY POINT IN THE ANALYSIS
C      MBO - BIGGEST BACKORDER IN THE MODEL EQUATION, LEFT OR RIGHT,
C           AFTER MODEL IN EXPANDED FORM. MAXIMUM ALLOWED IS 200
C     *MCSE - SET = 0 DO NOT WISH STANDARD ERRORS OF RESIDUAL AUTOCORRELATIONS.
C            ANY OTHER VALUE STANDARD ERRORS ARE CALCULATED
C      MD - NO PURPOSE HERE
C     *MFAC - ARRAY OF SIZE 3.  MFAC(1)= NO. OF AUTOREGRESSIVE FACTORS IN TIME
C            SERIES MODEL, MFAC(2)= NO. OF DIFFERENCE FACTORS IN TIME SERIES
C            MODEL, MFAC(3)= NO. OF MOVING AVERAGE FACTORS IN TIME SERIES MODEL
C     *MIT - MAXIMUM NUMBER OF ITERATIONS FOR ESTIMATION.
C           MAXIMUM ALLOWED IS 999
C      MPRINT - NO PURPOSE HERE.  IS SET=1 BY A SUBROUTINE
C     *MPROB - SET SO THAT IS NOT = NPROB
C     *NAC - NUMBER OF AUTOCORRELATIONS OF RESIDUALS
C     *NAPL - NUMBER OF RESIDUAL AUTOCORRELATIONS PRINTED PER LINE.
C            BETWEEN 1 AND 12 INCLUSIVE
C     *NCHI - THE NUMBER OF AUTOCORRELATIONS TO BE USED IN CALCULATING A
C            CHI-SQUARE STATISTIC.  SET LESS THAN OR = 0 IF NOT WANTED.
C            MAXIMUM VALUE IS NAC
C     *ND - ARRAY OF MINIMUM SIZE MFAC(2).  CONTAINS THE MFAC(2) NUMBERS OF
C          DIFFERENCES OF EACH TYPE OF DIFFERENCE DESIRED IN TIME SERIES MODEL.
C          (SEE IOD)
C      NDIFAC - NO PURPOSE HERE.  IS SET=0 BY A SUBROUTINE
C     *NDIMS - PRESUMED REQUIRED SPACE IN THE TEMPORARY STORAGE ARRAY SCRATC.
C             MUST BE THE EXPRESSION GIVEN
C     *NF - NUMBER OF FORECASTS DESIRED. BETWEEN 0 AND 300 INCLUSIVE
C     *NMODEL - THE NUMBER OF SEPERATE MODELS TO BE ESTIMATED OR FORECAST
C              FOR A SINGLE TIME SERIES PROVIDED
C     *NOB - NUMBER OF OBSERVATIONS IN TIME SERIES TO BE MODELLED OR FORECAST
C      NP - NUMBER OF PARAMETERS IN THE MODEL. MAXIMUM ALLOWED IS 50
C     *NPAC - NUMBER OF PARTIAL AUTOCORRELATIONS OF RESIDUALS
C            MAXIMUM VALUE IS NAC
C     *NPROB - MODEL OR PROBLEM NUMBER PRINTED WITH OUTPUT
C             SOME GRAPH TITLES NOT GOOD IF EXCEEDS 9
```

```
C     *NT - ARRAY OF MINIMUM SIZE NTO, CONTAINING FORECAST TIME ORIGINS.
C           EACH ORIGIN MUST BE AT LEAST MBO AND NOT MORE THAN NOB
C     *NTO - NUMBER OF TIME ORIGINS FOR FORECASTS.
C            SOME GRAPH TITLES NOT GOOD IF EXCEEDS 9
C     *NU - NUMBER OF NEW OBSERVATIONS READ IN FOR UPDATES OF FORECASTS,
C           AND NUMBER OF UPDATES PRODUCED. MAXIMUM VALUE IS NF-1
C     *PA - ARRAY OF MINIMUM SIZE NP. FOR ESTIMATION, CONTAINS INITIAL
C           ESTIMATES OF PARAMETERS (NON-ZERO). FOR FORECASTING, CONTAINS
C           PREVIOUSLY ESTIMATED PARAMETER VALUES (AUTOMATICALLY THERE IF
C           ESTIMATION DONE FIRST)
C      PHI(NPAC,1) - CONTAINS PARTIAL AUTOCORRELATIONS OF RESIDUALS
C      RHO(NAC,1) - CONTAINS AUTOCORRELATIONS OF RESIDUALS
C      SCRATC - TEMPORARY STORAGE ARRAY WITH REQUIRED DIMENSION GREATER THAN OR =
C               NDIMS
C     *SERIES - ARRAY OF SIZE 20. CONTAINS TITLE DESCRIBING DATA OR ANALYSIS
C      SM - STANDARD DEVIATION OF RESIDUALS
C      STE(NAC,1) - CONTAINS ST. ERRORS OF RESIDUAL AUTOCORRELATIONS
C      TITL - ARRAY OF SIZE 20. NO PURPOSE HERE.
C     *TLAM - DATA TRANSFORMATION PARAMETER.  IF TLAM=1.0, THE SERIES Z REMAINS
C             AS READ IN.  IF TLAM=0, THE TRANSFORMED SERIES IS LN(Z+TM).  FOR
C             ANY OTHER VALUE OF TLAM, THE TRANSFORMED SERIES IS (Z+TM)**TLAM.
C             (SEE TM)
C     *TM - DATA TRANSFORMATION PARAMETER.  IF TRANSFORMATION IS REQUESTED
C           (SEE TLAM), THIS VALUE IS ADDED TO EACH VALUE OF THE SERIES Z BEFORE
C           TRANSFORMATION IS MADE
C      U - ARRAY OF MINIMUM SIZE NF. CONTAINS UPDATES FOR LAST NEW OBSERVATION
C          IN POSITIONS NU+1 THROUGH NF. FOR POSITIONS J=2,3,...,NU, CONTAINS
C          UPDATE OF ORIGINAL J PERIOD AHEAD FORECAST ASSUMING ALL PREVIOUS
C          OBSERVATIONS KNOWN. POSITION 1 IS EMPTY
C      VLAM - NO PURPOSE HERE
C      VM - NO PURPOSE HERE
C     *Z - ARRAY FOR ORIGINAL TIME SERIES DATA. MINIMUM SIZE IS NOB+NF
C     *ZN - ARRAY FOR NEW OBSERVATIONS USED IN UPDATING FORECASTS.
C           MINIMUM SIZE IS NU
```

```
C     ZP(NF,NTO) - ARRAY CONTAINING THE NF TIME SERIES FORECASTS AT THE J TH
C                  TIME ORIGIN IN ITS J TH COLUMN
C
C     OUTPUT OF THIS EXAMPLE
C     **********************
C
C     FOR A SERIES OF 157 OBSERVATIONS, ESTIMATION AND FORECASTING WILL BE
C     DONE FOR ONE MODEL.  THE DATA, TITLED 'U S TREASURY BILLS INTEREST RATE,
C     MONTHLY JANUARY 1956 THROUGH 4ANUARY 1969', IS READ INTO THE PROGRAM BY A
C     20F4.3 FORMAT SPECIFICATION.  THE DATA READ IN IS USED IN THE ANALYSIS,
C     SINCE THE TRANSFORMATION PARAMETER TLAM=1.0.  THE MODEL USED CONTAINS ONE
C     DIFFERENCE FACTOR, WITH ONE DIFFERENCE OF ORDER 1, I.E. (1-B**1)**1 =
C     (1-B).  IT ALSO CONTAINS ONE AUTOREGRESSIVE FACTOR, BUT NO MOVING
C     AVERAGE FACTORS.  THE AUTOREGRESSIVE FACTOR CONTAINS 2 PARAMETERS, WITH
C     ORDERS 1 AND 2, RESPECTIVELY.  THE INITIAL VALUES OF THESE PARAMETERS
C     ARE .8 AND .5.  THE MODEL DOES NOT HAVE A MEAN OR TREND PARAMETER.
C     THUS, INITIALLY THE MODEL COULD BE WRITTEN -
C
C          (1-.8B-.5B**2)*(1-B)*Z(T) = A(T)
C
C     THE ITERATION WILL BE STOPPED WHEN THE RELATIVE CHANGE IN EACH PARAMETER
C     IS LESS THAN .004, OR WHEN A MAXIMUM OF 100 ITERATIONS IS REACHED.  THE
C     ESTIMATION ROUTINE WILL PLOT THE ESTIMATED RESIDUALS.  IT WILL NOT PLOT
C     OR LIST THE ORIGINAL DATA.  24 AUTOCORRELATIONS AND PARTIAL AUTOCORR-
C     ELATIONS WILL BE CALCULATED FROM THE RESIDUALS, PRINTED 8 PER LINE, AND
C     PLOTTED.  STANDARD ERRORS OF THESE AUTOCORRELATIONS WILL BE CALCULATED.
C     20 FORECASTS WILL BE CALCULATED AT EACH OF 2 TIME ORIGINS, T=85 AND
C     T=157.  NO UPDATED FORECASTS ARE TO BE CALCULATED.  THE 95 PER CENT
C     CONFIDENCE LIMITS WILL BE OUTPUT WITH THE FORECASTS.  THE FORECASTING
C     ROUTINE WILL PLOT THE FORECASTS.  IT WILL NOT PLOT OR LIST THE
C     ORIGINAL DATA.
C
C     SPECIAL NOTE
C     ************
```

```
C
C     THE STATEMENTS BEGINNING WITH 'COMMON' MUST BE INCLUDED IN THE PROGRAM.
C     VARIABLE NAMES MAY BE CHANGED, BUT NOT THE NAME BETWEEN THE SLASH MARKS.
```

```
296
(16F5.1)
 53.8 53.6 53.5 53.5 53.4 53.1 52.7 52.4 52.2 52.0 52.0 52.4 53.0 54.0 54.9 56.0
 56.8 56.8 56.4 55.7 55.0 54.3 53.2 52.3 51.6 51.2 50.8 50.5 50.0 49.2 48.4 47.9
 47.6 47.5 47.5 47.6 48.1 49.0 50.0 51.1 51.8 51.9 51.7 51.2 50.0 48.3 47.0 45.8
 45.6 46.0 46.9 47.8 48.2 48.3 47.9 47.2 47.2 48.1 49.4 50.6 51.5 51.6 51.2 50.5
 50.1 49.8 49.6 49.4 49.3 49.2 49.3 49.7 50.3 51.3 52.8 54.4 56.0 56.9 57.5 57.3
 56.6 56.0 55.4 55.4 56.4 57.2 58.0 58.4 58.4 58.1 57.7 57.0 56.0 54.7 53.2 52.1
 51.6 51.0 50.5 50.4 51.0 51.8 52.4 53.0 53.4 53.6 53.7 53.8 53.8 53.8 53.3 53.0
 52.9 53.4 54.6 56.4 58.0 59.4 60.2 60.0 59.4 58.4 57.6 56.9 56.4 56.0 55.7 55.3
 55.0 54.4 53.7 52.8 51.6 50.6 49.4 48.8 48.5 48.7 49.2 49.8 50.4 50.7 50.9 50.7
 50.5 50.4 50.2 50.4 51.2 52.3 53.2 53.9 54.1 54.0 53.6 53.2 53.0 52.8 52.3 51.9
 51.6 51.6 51.4 51.2 50.7 50.0 49.4 49.3 49.7 50.6 51.8 53.0 54.0 55.3 55.9 55.9
 54.6 53.5 52.4 52.1 52.3 53.0 53.8 54.6 55.4 55.9 55.9 55.2 54.4 53.7 53.6 53.6
 53.2 52.5 52.0 51.4 51.0 50.9 52.4 53.5 55.6 58.0 59.5 60.0 60.4 60.5 60.2 59.7
 59.0 57.6 56.4 55.2 54.5 54.1 54.1 54.4 55.5 56.2 57.0 57.3 57.4 57.0 56.4 55.9
 55.5 55.3 55.2 55.4 56.0 56.5 57.1 57.3 56.8 55.6 55.0 54.1 54.3 55.3 56.4 57.2
 57.8 58.3 58.6 58.8 58.8 58.6 58.0 57.4 57.0 56.4 56.3 56.4 56.4 56.0 55.2 54.0
 53.0 52.0 51.6 51.6 51.1 50.4 50.0 50.0 52.0 54.0 55.1 54.5 52.8 51.4 50.8 51.2
 52.0 52.8 53.8 54.5 54.9 54.9 54.8 54.4 53.7 53.3 52.8 52.6 52.6 53.0 54.3 56.0
 57.0 58.0 58.6 58.5 58.3 57.8 57.3 57.0
PER CENT CARBON DIOXIDE IN OUTLET GAS
(10F8.3)
   -.109    .000    .178    .339    .373    .441    .461    .348    .127   -.180
   -.588  -1.055  -1.421  -1.520  -1.302   -.814   -.475   -.193    .088    .435
    .771    .866    .875    .891    .987   1.263   1.775   1.976   1.934   1.866
   1.832   1.767   1.608   1.265    .790    .360    .115    .088    .331    .645
    .960   1.409   2.670   2.834   2.812   2.483   1.929   1.485   1.214   1.239
   1.608   1.905   2.023   1.815    .535    .122    .009    .164    .671   1.019
   1.146   1.155   1.112   1.121   1.223   1.257   1.157    .913    .620    .255
   -.280  -1.080  -1.551  -1.799  -1.825  -1.456   -.944   -.570   -.431   -.577
   -.960  -1.616  -1.875  -1.891  -1.746  -1.474  -1.201   -.927   -.524    .040
    .788    .943    .930   1.006   1.137   1.198   1.054    .595   -.080   -.314
   -.288   -.153   -.109   -.187   -.255   -.229   -.007    .254    .330    .102
```

-.423	-1.139	-2.275	-2.594	-2.716	-2.510	-1.790	-1.346	-1.081	-.910
-.876	-.885	-.800	-.544	-.416	-.271	.000	.403	.841	1.285
1.607	1.746	1.683	1.485	.993	.648	.577	.577	.632	.747
.900	.993	.968	.790	.399	-.161	-.553	-.603	-.424	-.194
-.049	.060	.161	.301	.517	.566	.560	.573	.592	.671
.933	1.337	1.460	1.353	.772	.218	-.237	-.714	-1.099	-1.269
-1.175	-.676	.033	.556	.643	.484	.109	-.310	-.697	-1.047
-1.218	-1.183	-.873	-.336	.063	.084	.000	.001	.209	.556
.782	.858	.918	.862	.416	-.336	-.959	-1.813	-2.378	-2.499
-2.473	-2.330	-2.053	-1.739	-1.261	-.569	-.137	-.024	-.050	-.135
-.276	-.534	-.871	-1.243	-1.439	-1.422	-1.175	-.813	-.634	-.582
-.625	-.713	-.848	-1.039	-1.346	-1.628	-1.619	-1.149	-.488	-.160
-.007	-.092	-.620	-1.086	-1.525	-1.858	-2.029	-2.024	-1.961	-1.952
-1.794	-1.302	-1.030	-.918	-.798	-.867	-1.047	-1.123	-.876	-.395
.185	.662	.709	.605	.501	.603	.943	1.223	1.249	.824
.102	.025	.382	.922	1.032	.866	.527	.093	-.458	-.748
-.947	-1.029	-.928	-.645	-.424	-.276	-.158	-.033	.102	.251
.280	.000	-.493	-.759	-.824	-.740	-.528	-.204	.034	.204
.253	.195	.131	.017	-.182	-.262				

CODED INPUT GAS RATE IN CUBIC FEET PER MINUTE

```
   0      0      0      0
1.0000  .0000    1      0      0
   3      1      0
   1      2      3      0
1.9749 -1.3732  .3424  -.0610
1.0000  .0000    0
  20      1     20      2      2
   1
   1      1
  20      0     10     20
   0      1      0      1      1
```

```
C     COMMENTS ON TRANSFER FUNCTION IDENTIFICATION EXAMPLE
C
C      HOW DIMENSIONS RESTRICT PROBLEM
C      *******************************
C
C          25 PARAMETERS IN THE PREWHITENING TRANSFORMATION
C          150 AUTOCORRELATIONS ON RELEVANT SERIES
C          150 PARTIAL AUTOCORRELATIONS ON RELEVANT SERIES
C          2 REGULAR DIFFERENCES ON RELEVANT SERIES IN AUTOCORRELATION CALCULATION
C          500 OBSERVATIONS IN INPUT SERIES, AND IN OUTPUT SERIES
C          54 MAXIMUM LAG IN CROSS CORRELATION CALCULATIONS
C          55 IMPULSE RESPONSE WEIGHTS ESTIMATED
C
C      LIST OF VARIABLES (* MEANS USER SUPPLIED)
C      *****************************************
C
C      A - ARRAY OF OUTPUT SERIES TRANSFORMED BY APPLICATION OF THE INPUT
C          PREWHITENING TRANSFORMATION. MINIMUM SIZE IS NOB.
C      B - ARRAY OF PREWHITENED INPUT SERIES. MINIMUM SIZE IS NOB.
C      E - ARRAY CONTAINING MEAN OF RELEVANT SERIES AND REQUESTED DIFFERENCES.
C      EPS1 - NO PURPOSE IN TRANSFER FUNCTION IDENTIFICATION.
C      EPS2 - NO PURPOSE IN TRANSFER FUNCTION IDENTIFICATION.
C     *FMX - ARRAY OF SIZE 20.  CONTAINS THE FORMAT SPECIFICATION FOR
C            INPUT TO PROGRAM OF INPUT TIME SERIES.
C     *FMY - ARRAY OF SIZE 20.  CONTAINS THE FORMAT SPECIFICATION FOR
C            INPUT TO PROGRAM OF OUTPUT TIME SERIES.
C     *ICALCF - ARRAY OF SIZE 5. SET EACH ELEMENT = 0 TO SUPPRESS CALCULATION
C               OF VARIOUS CORRELATION FUNCTIONS. THE FOLLOWING CORRELATION
C               FUNCTIONS WILL BE CALCULATED IF THE CORRESPONDING ELEMENT IN
C               THIS ARRAY IS NOT ZERO:
C
C                   ICALCF(1) : AUTOCORRELATIONS OF THE OUTPUT SERIES
C                   ICALCF(2) : AUTOCORRELATIONS OF THE INPUT SERIES
C                   ICALCF(3) : CROSS CORRELATIONS OF THE INPUT SERIES WITH THE
```

```
C                              OUTPUT SERIES
C                  ICALCF(4) : AUTOCORRELATIONS OF THE PREWHITENED OUTPUT SERIES
C                  ICALCF(5) : AUTOCORRELATIONS OF THE PREWHITENED INPUT SERIES
C    *ILDCC - CONTROLS LISTING OF DATA.  VALUES ASSIGNED IN SAME WAY AS IPDCC.
C    *ILDID - SET = 0 TO SUPPRESS LISTING OF RELEVANT DATA BY AUTOCORRELATION
C             ROUTINE.
C     ILDEST - NO PURPOSE FOR TRANSFER FUNCTION IDENTIFICATION.
C    *INC - ARRAY CONTAINING (MFAC(1)+MFAC(3)+2) NUMBERS OF EACH OF THE
C           SPECIFIED TYPES OF PARAMETERS IN THE PREWHITENING MODEL FORM.
C     IOD - ARRAY THAT AT VARIOUS TIMES IN THE ANALYSIS WILL CONTAIN CONTENTS
C           OF EITHER IODY OR IODX.
C    *IODX - SIMILAR TO IODY, BUT FOR INPUT SERIES.
C    *IODY - ARRAY OF MINIMUM SIZE MFACTM(2).  CONTAINS THE MFACTM(2) ORDERS OF
C            DIFFERENCES OF EACH TYPE OF DIFFERENCE DESIRED IN PREWHITENING
C            MODEL FOR OUTPUT SERIES.
C    *IOPA - ARRAY OF MINIMUM SIZE NP. CONTAINS THE ORDER OF EACH PARAMETER FROM
C            LEFT TO RIGHT IN THE MODEL FOR THE PREWHITENING TRANSFORMATION
C            (POWER OF B OPERATOR).
C    *IPDCC - CONTROLS OUTPUT, INPUT SERIES PLOTTING.  SET AS FOLLOWS:
C                   IPDCC=0 : NEITHER PLOTTED
    2  FORMAT(16I5)                                                       50210
    4  FORMAT(20A4)                                                       50220
   12  FORMAT(10F8.4)                                                     50230
   26  FORMAT(2F8.4,4I5)                                                  50240
       END                                                                50250
```

APPENDIX E

GUIDE TO THE USE OF TIME SERIES ANALYSIS TECHNIQUES

APPENDIX E

GUIDE TO THE USE OF TIME SERIES ANALYSIS TECHNIQUES

This appendix is designed to assist in the use of the computer programs on the CREATE system that constitute the Time Series Analysis package. These programs are in the AFIT.LIB and are run in the CARDIN system. This appendix is divided into five sections: (1) building input files; (2) the use of AFIT.LIB/UNIDEN,R for model identification; (3) guides to model identification; (4) estimation of parameters; and (5) the use of AFIT.LIB/UNEST,R for parameter estimation and forecasting.

Building Input Files

A great deal of difficulty was initially encountered in attempting to build files that could be read by the Box and Jenkins time series analysis programs. The difficulty was overcome by examining the main programs to determine data format and placement requirements and then by building a computer program to create files in the required format. The terms "create a file" were abbreviated to the computer program's name CAFILE which can be found under SL.LIB/CAFILE,R. The file read by the time series analysis programs contains both parameters and data elements. The data elements can be entered in two ways in

the CAFILE program: (1) directly from the terminal, or (2) by using a random generator. The parameters in the file are dependent upon which time series computer program the file is being built for. Many of the parameters are the same from program to program but are placed differently in CAFILE for each program.

CAFILE can be used to build files for input into AFIT.LIB/UNIDEN,R and AFIT.LIB/UNEST,R. A series of questions in the program insure the correct placement of the parameters in the file. The first few times in using CAFILE, it is recommended that assistance be requested when the program asks "Do you want instructions on file building?" Explanations of such terms as TLAM, TM, NDIFAC, NAPI, NAC, NPAC, MCSE, ILPRINT, ND, IOD, NCHI, etc., are presented. After gaining familiarlity with time series analysis parameters, the shortened version may be used to build a file in a few minutes.

The CAFILE program is designed to handle 500 data elements, the capacity of the time series analysis programs.

Before using CAFILE, it is necessary to create a file name with enough space to put the output of CAFILE into. This can be easily accomplished using the ACCESS system to create file space on disk storage. If this is not accomplished, the program will "bomb."

The CAFILE program runs in the following manner:

1. The program asks if this is your first experience with time series analysis.

2. The program asks if you are building a file for forecasting. If the answer is "yes," then the file will be built for input into AFIT/UNEST,R. If the answer is "no," then a file will be built for input into AFIT/UNIDEN,R.

3. The program asks for the number of observations and what format is desired. If a random generator is used to create data elements, the most desirable format is (v).

4. The program asks for the file name where the data will be written. As mentioned earlier if the file is not available, the program will "bomb."

5. The program asks for the values of the specific parameters required by the time series program.

6. The program asks if data will be entered from the terminal or from a random generator. If a random generator is asked for, the following choices are available:

a. A Poisson process generator with a mean of ten.

b. A linearly increasing Poisson process generator. The mean is initially set at 10 and then is incremented by one-twelfth for each newly generated data element.

c. A linearly decreasing Poisson process generator. The mean is initially set at 40 and then is decremented by one-sixth for each newly generated data element.

d. An alternating linear Poisson process generator. The mean increases for the first twelve data elements and then decreases for the next six data elements and then continues to increase and decrease in the same manner to the 120th data element, thereafter continuing to increase with each successive data element.

e. A sine process generator. The equation is: 20Sine(Poisson)+50.

7. After generating the data elements or receiving them from the terminal, the program will ask if a forecasting file is being built. If the answer is "yes," the file will be written to the previously designated file in the proper format for input into AFIT/UNEST,R. If the answer is "no," the file will be written to the previously designated file in the proper format for input into AFIT/UNIDEN,R.

8. The program will then ask if you are building another file. If the answer is "yes," the program returns to the beginning. A word of caution: *should a file using a random generator need to be replicated, it must be done exactly the same way as the original.* The easiest way to

insure replicability is to return to the * level and "RUN" the program again (see the program contained in Appendix C).

The Use of AFIT.LIB/UNIDEN,R for Model Identification

The dimension limitations of the UNIDEN program are:

500 Observations

150 Autocorrelations

150 Partial Autocorrelations

2 Autocorrelation functions of differences of the original series.

The variables in the program which must be supplied by the user are:

FMZ The format specification (max size = 20).

ILDID The data list. Set = 0 to suppress data.

IOD An array containing NDIFAC orders of differences (d) of each type desired, i.e., the values of s in the expression $(1-B^s)$.

IPDID The plotting data function in the program. Set = 0 to suppress.

IWTPA The autocorrelation plotting function of the program. Set = 0 to suppress.

MCSE The standard error of autocorrelation calculating function. Set = 0 to suppress calculation.

MPRINT A function for output of the statistics in the program. Set = 0 to suppress listing.

NAC The number of autocorrelations.

NAPL The number of autocorrelations to be printed per line, between 1 and 12 inclusive.

NCHI The number of autocorrelations to be used in calculating a chi-square statistic. Set = 0 if not desired. Obviously, the maximum dimension of NCHI is NAC.

ND An array containing the NDIFAC numbers of differences of each type desired. The autocorrelation function will be calculated for the ND(1)+1 series. ND(2), NC(3), etc., are used to form another "original" series from the data matrix entered in G. The minimum size is the value of NADIFAL. Warning: *Some graph titles will not be right if ND(1) exceeds five.*

NDIFAC The number of difference factors or types. The autocorrelation function is calculated for the original series and each requested difference of type 1. Any

difference factors beyond one are used to difference the original series to form a new "original" series.

NOB The number of observations.

NPAC The number of partial autocorrelations. The maximum value is NAC.

SERIES An array of 80 spaces to place the title describing the data or the analysis.

TLAM A data transformation parameter. If TLAM is set equal to zero, the original data elements will be transformed into a series with $\ln(g_i+TM)$, $i=1,2,\ldots,NOB$ as the data elements. If TLAM is set equal to one, the data elements are left as input. Any values greater than one, the series will be transformed as follows: $(g_i+TM)^{TLAM}$, $i=1,2,\ldots,NOB$.

TM A data transformation parameter which is added to each data element before it is transformed in accordance with TLAM.

In using CAFILE, much of the above information is repeated with each parameter if assistance is requested when the program asks, "Do you want instructions on file building?" Output of UNIDEN of the three series is contained in Tables E.1 to E.9 and Figures E.1 to E.15.

TABLE E.1

OBSERVATIONS OF A LINEARLY INCREASING SERIES

1- 8	0.130000E 02	0.100000E 02	0.700000E 01	0.800000E 01	0.700000E 01	0.800000E 01	0.900000E 01	0.100000E 02
9- 16	0.100000E 02	0.160000E 02	0.110000E 02	0.120000E 02	0.110000E 02	0.130000E 02	0.600000E 01	0.800000E 01
17- 24	0.140000E 02	0.900000E 01	0.120000E 02	0.170000E 02	0.130000E 02	0.110000E 02	0.140000E 02	0.130000E 02
25- 32	0.160000E 02	0.900000E 01	0.150000E 02	0.700000E 01	0.150000E 02	0.500000E 01	0.130000E 02	0.130000E 02
33- 40	0.130000E 02	0.120000E 02	0.130000E 02	0.700000E 01	0.180000E 02	0.100000E 02	0.120000E 02	0.200000E 02
41- 48	0.700000E 01	0.600000E 01	0.150000E 02	0.210000E 02	0.160000E 02	0.140000E 02	0.120000E 02	0.160000E 02
49- 56	0.110000E 02	0.140000E 02	0.200000E 02	0.160000E 02	0.160000E 02	0.170000E 02	0.140000E 02	0.190000E 02
57- 64	0.160000E 02	0.150000E 02	0.120000E 02	0.120000E 02	0.120000E 02	0.190000E 02	0.190000E 02	0.130000E 02
65- 72	0.180000E 02	0.100000E 02	0.160000E 02	0.900000E 01	0.150000E 02	0.180000E 02	0.180000E 02	0.200000E 02
73- 80	0.180000E 02	0.190000E 02	0.160000E 02	0.900000E 01	0.140000E 02	0.170000E 02	0.190000E 02	0.200000E 02
81- 88	0.190000E 02	0.170000E 02	0.180000E 02	0.110000E 02	0.140000E 02	0.130000E 02	0.130000E 02	0.160000E 02
89- 96	0.170000E 02	0.130000E 02	0.190000E 02	0.150000E 02	0.180000E 02	0.220000E 02	0.180000E 02	0.210000E 02
97- 104	0.200000E 02	0.170000E 02	0.260000E 02	0.220000E 02	0.120000E 02	0.180000E 02	0.160000E 02	0.190000E 02
105- 112	0.150000E 02	0.250000E 02	0.900000E 01	0.150000E 02	0.150000E 02	0.150000E 02	0.130000E 02	0.190000E 02
113- 120	0.220000E 02	0.150000E 02	0.170000E 02	0.200000E 02	0.250000E 02	0.190000E 02	0.200000E 02	0.270000E 02

TABLE E.2

AUTOCORRELATION OF LINEARLY INCREASING SERIES

ORIGINAL SERIES
MEAN OF THE SERIES =0.14775E 02
ST. DEV. OF SERIES =0.45123E 01
NUMBER OF OBSERVATIONS = 120

1- 12	0.37	0.38	0.34	0.28	0.18	0.28	0.28	0.28	0.18	0.19	0.26	0.17
ST.E.	0.09	0.10	0.11	0.12	0.13	0.13	0.13	0.14	0.14	0.15	0.15	0.15
13- 24	0.17	0.20	0.15	0.17	0.27	0.22	0.28	0.22	0.23	0.13	0.24	0.20
ST.E.	0.15	0.15	0.16	0.16	0.16	0.16	0.17	0.17	0.17	0.17	0.18	0.18
25- 36	0.22	0.21	0.20	0.06	0.07	0.06	0.05	0.04	0.02	0.10	0.07	0.00
ST.E.	0.18	0.18	0.18	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19

MEAN DIVIDED BY ST. ERROR = 0.35869E 02

TO TEST WHETHER THIS SERIES IS WHITE NOISE, THE VALUE 0.19218E 03
SHOULD BE COMPARED WITH A CHI-SQUARE VARIABLE WITH 36 DEGREES OF FREEDOM

DIFFERENCE 1
MEAN OF THE SERIES =0.11765E 00
ST. DEV. OF SERIES =0.49646E 01
NUMBER OF OBSERVATIONS = 119

1- 12	-0.51	0.04	-0.01	0.06	-0.15	0.08	-0.03	0.10	-0.06	-0.07	0.14	-0.08
ST.E.	0.09	0.11	0.11	0.11	0.11	0.11	0.12	0.12	0.12	0.12	0.12	0.12
13- 24	0.01	-0.01	-0.02	-0.08	0.14	-0.10	0.13	-0.10	0.07	-0.13	0.11	-0.05
ST.E.	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.13
25- 36	0.03	-0.02	0.12	-0.10	-0.01	0.03	-0.01	0.00	-0.08	0.11	0.01	-0.08
ST.E.	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13

MEAN DIVIDED BY ST. ERROR = 0.25851E 00

TABLE E.3

PARTIAL AUTOCORRELATION FUNCTION OF A LINEARLY INCREASING SERIES

PARTIAL AUTOCORRELATIONS

DATA - ILINPOI; DISTRIBUTION 10 YEARS DATA 120 OBSERVATIONS

DIFFERENCING - ORIGINAL SERIES IS YOUR DATA.

DIFFERENCES BELOW ARE OF ORDER 1

ORIGINAL SERIES
MEAN OF THE SERIES =0.14775E 02
ST. DEV. OF SERIES =0.45123E 01
NUMBER OF OBSERVATIONS = 120

1- 12	0.37	0.28	0.17	0.08	-0.05	0.13	0.14	0.11	-0.07	-0.04	0.15	0.01
13- 24	-0.00	-0.00	-0.03	0.08	0.18	0.02	0.06	-0.01	0.04	-0.07	0.11	0.02
25- 36	-0.01	0.04	-0.01	-0.17	-0.07	-0.04	-0.02	-0.03	-0.08	0.05	0.03	-0.08

DIFFERENCE 1
MEAN OF THE SERIES =0.11765E 00
ST. DEV. OF SERIES =0.49646E 01
NUMBER OF OBSERVATIONS = 119

1- 12	-0.51	-0.29	-0.19	-0.04	-0.19	-0.15	-0.14	0.03	0.05	-0.12	0.05	0.02
13- 24	0.05	0.01	-0.08	-0.20	-0.05	-0.08	0.04	-0.05	0.01	-0.13	-0.03	0.00
25- 36	-0.04	-0.03	0.16	0.11	0.04	-0.00	-0.01	0.04	-0.05	-0.01	0.07	-0.03

TABLE E.4

OBSERVATIONS OF A LINEARLY DECREASING SERIES

DATA - DLINPOI) DISTRIBUTION 10 YEARS DATA   120 OBSERVATIONS

1- 8	0.380000E 02	0.300000E 02	0.490000E 02	0.420000E 02	0.340000E 02	0.480000E 02	0.400000E 02	0.430000E 02
9- 16	0.320000E 02	0.380000E 02	0.310000E 02	0.400000E 02	0.330000E 02	0.440000E 02	0.350000E 02	0.390000E 02
17- 24	0.450000E 02	0.360000E 02	0.410000E 02	0.330000E 02	0.310000E 02	0.410000E 02	0.370000E 02	0.280000E 02
25- 32	0.360000E 02	0.430000E 02	0.400000E 02	0.270000E 02	0.340000E 02	0.420000E 02	0.340000E 02	0.320000E 02
33- 40	0.230000E 02	0.350000E 02	0.240000E 02	0.310000E 02	0.380000E 02	0.390000E 02	0.350000E 02	0.430000E 02
41- 48	0.250000E 02	0.330000E 02	0.290000E 02	0.350000E 02	0.230000E 02	0.200000E 02	0.320000E 02	0.300000E 02
49- 56	0.290000E 02	0.280000E 02	0.380000E 02	0.330000E 02	0.340000E 02	0.290000E 02	0.290000E 02	0.390000E 02
57- 64	0.250000E 02	0.250000E 02	0.320000E 02	0.250000E 02	0.290000E 02	0.300000E 02	0.210000E 02	0.330000E 02
65- 72	0.320000E 02	0.310000E 02	0.200000E 02	0.300000E 02	0.330000E 02	0.250000E 02	0.280000E 02	0.240000E 02
73- 80	0.290000E 02	0.240000E 02	0.250000E 02	0.350000E 02	0.270000E 02	0.310000E 02	0.270000E 02	0.390000E 02
81- 88	0.350000E 02	0.320000E 02	0.260000E 02	0.320000E 02	0.240000E 02	0.230000E 02	0.190000E 02	0.250000E 02
89- 96	0.260000E 02	0.300000E 02	0.270000E 02	0.240000E 02	0.300000E 02	0.230000E 02	0.190000E 02	0.260000E 02
97- 104	0.240000E 02	0.130000E 02	0.240000E 02	0.270000E 02	0.250000E 02	0.290000E 02	0.330000E 02	0.210000E 02
105- 112	0.210000E 02	0.270000E 02	0.260000E 02	0.170000E 02	0.240000E 02	0.210000E 02	0.320000E 02	0.230000E 02
113- 120	0.260000E 02	0.250000E 02	0.210000E 02	0.170000E 02	0.200000E 02	0.180000E 02	0.210000E 02	0.200000E 02

TABLE E.5

AUTOCORRELATION FUNCTION OF A LINEARLY DECREASING SERIES

ORIGINAL SERIES
MEAN OF THE SERIES =0.30050E 02
ST. DEV. OF SERIES =0.72574E 01
NUMBER OF OBSERVATIONS = 120

1- 12	0.49	0.49	0.49	0.48	0.34	0.28	0.33	0.38	0.38	0.39	0.41	0.41
ST.E.	0.09	0.11	0.13	0.14	0.16	0.16	0.17	0.17	0.18	0.18	0.19	0.20
13- 24	0.40	0.31	0.28	0.22	0.25	0.22	0.20	0.24	0.25	0.30	0.17	0.22
ST.E.	0.20	0.21	0.22	0.22	0.22	0.22	0.22	0.23	0.23	0.23	0.23	0.23
25- 36	0.21	0.20	0.10	0.12	0.10	0.10	0.11	0.06	0.07	0.10	0.02	0.01
ST.E.	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24

MEAN DIVIDED BY ST. ERROR = 0.45358E 02

TO TEST WHETHER THIS SERIES IS WHITE NOISE, THE VALUE 0.36003E 03
SHOULD BE COMPARED WITH A CHI-SQUARE VARIABLE WITH 36 DEGREES OF FREEDOM

DIFFERENCE 1
MEAN OF THE SERIES =0.15126E 00
ST. DEV. OF SERIES =0.72960E 01
NUMBER OF OBSERVATIONS = 119

1- 12	-0.51	-0.02	0.01	0.15	-0.10	-0.09	-0.01	0.06	-0.01	-0.01	0.02	-0.01
ST.E.	0.09	0.11	0.11	0.11	0.11	0.12	0.12	0.12	0.12	0.12	0.12	0.12
13- 24	0.10	-0.05	0.01	-0.10	0.10	-0.03	-0.05	0.03	-0.05	0.18	-0.16	0.05
ST.E.	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
25- 36	-0.01	0.09	-0.09	0.01	-0.02	0.00	0.06	-0.05	-0.04	0.13	-0.08	-0.07
ST.E.	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.13	0.13

MEAN DIVIDED BY ST. ERROR = 0.22616E 00

TABLE E.6

PARTIAL AUTOCORRELATION OF A LINEARLY DECREASING SERIES

PARTIAL AUTOCORRELATIONS

DATA - DLINPOIJ DISTRIBUTION 10 YEARS DATA　　　120 OBSERVATIONS

DIFFERENCING - ORIGINAL SERIES IS YOUR DATA.

DIFFERENCES BELOW ARE OF ORDER 1

ORIGINAL SERIES
MEAN OF THE SERIES =0.30050E 02
ST. DEV. OF SERIES =0.72574E 01
NUMBER OF OBSERVATIONS = 120

1- 12	0.49	0.33	0.25	0.19	-0.07	-0.11	0.06	0.20	0.20	0.16	0.06	-0.03
13- 24	0.00	-0.10	-0.07	-0.08	0.06	0.07	0.01	0.00	-0.05	0.07	-0.16	-0.02
25- 36	0.03	0.08	-0.01	-0.03	-0.09	-0.08	0.05	-0.06	-0.03	0.04	-0.11	-0.03

DIFFERENCE 1
MEAN OF THE SERIES =0.15126E 00
ST. DEV. OF SERIES =0.72960E 01
NUMBER OF OBSERVATIONS = 119

1- 12	-0.51	-0.37	-0.30	-0.02	0.02	-0.12	-0.26	-0.27	-0.23	-0.16	-0.09	-0.14
13- 24	-0.03	0.01	0.07	-0.08	-0.08	-0.06	-0.05	0.03	-0.13	0.05	-0.06	-0.06
25- 36	-0.14	-0.08	-0.05	-0.00	0.01	-0.11	-0.00	0.02	-0.04	0.14	0.07	-0.06

TABLE E.7

OBSERVATIONS OF AN ALTERNATING LINEAR SERIES

DATA - RNDLIN; DISTRIBUTION 10 YEARS DATA 120 OBSERVATIONS

1- 8	0.240000E 02	0.160000E 02	0.130000E 02	0.190000E 02	0.260000E 02	0.260000E 02	0.240000E 02	0.160000E 02
9- 16	0.210000E 02	0.290000E 02	0.280000E 02	0.270000E 02	0.240000E 02	0.240000E 02	0.230000E 02	0.170000E 02
17- 24	0.220000E 02	0.220000E 02	0.160000E 02	0.250000E 02	0.230000E 02	0.210000E 02	0.180000E 02	0.330000E 02
25- 32	0.240000E 02	0.260000E 02	0.240000E 02	0.280000E 02	0.270000E 02	0.260000E 02	0.310000E 02	0.230000E 02
33- 40	0.260000E 02	0.220000E 02	0.290000E 02	0.230000F 02	0.230000E 02	0.190000E 02	0.240000E 02	0.290000E 02
41- 48	0.310000E 02	0.290000E 02	0.250000E 02	0.230000E 02	0.280000E 02	0.320000E 02	0.280000E 02	0.270000E 02
49- 56	0.200000E 02	0.240000E 02	0.230000E 02	0.180000E 02	0.280000E 02	0.240000E 02	0.310000E 02	0.300000E 02
57- 64	0.300000E 02	0.270000E 02	0.360000E 02	0.220000E 02	0.270000E 02	0.280000E 02	0.340000E 02	0.200000E 02
65- 72	0.230000E 02	0.250000E 02	0.260000E 02	0.290000E 02	0.270000E 02	0.330000E 02	0.250000E 02	0.310000E 02
73- 80	0.310000E 02	0.280000E 02	0.270000E 02	0.370000E 02	0.260000E 02	0.260000E 02	0.320000E 02	0.270000E 02
81- 88	0.280000E 02	0.300000E 02	0.290000E 02	0.340000E 02	0.330000E 02	0.300000E 02	0.200000E 02	0.370000E 02
89- 96	0.290000E 02	0.300000E 02	0.330000E 02	0.260000E 02	0.300000E 02	0.300000E 02	0.360000E 02	0.340000E 02
97- 104	0.320000E 02	0.500000E 02	0.460000E 02	0.350000E 02	0.350000E 02	0.360000E 02	0.320000E 02	0.260000E 02
105- 112	0.330000E 02	0.390000E 02	0.330000E 02	0.290000E 02	0.360000E 02	0.320000E 02	0.350000E 02	0.160000E 02
113- 120	0.330000E 02	0.420000E 02	0.480000E 02	0.370000E 02	0.420000E 02	0.360000E 02	0.330000E 02	0.390000E 02

TABLE E.8

AUTOCORRELATION OF AN ALTERNATING LINEAR SERIES

ORIGINAL SERIES
MEAN OF THE SERIES =0.28208E 02
ST. DEV. OF SERIES =0.65835E 01
NUMBER OF OBSERVATIONS = 120

1- 12	0.53	0.42	0.37	0.41	0.30	0.30	0.30	0.25	0.27	0.27	0.30	0.26
ST.E.	0.09	0.11	0.13	0.13	0.14	0.15	0.15	0.16	0.16	0.17	0.17	0.17
13- 24	0.31	0.30	0.38	0.36	0.36	0.29	0.26	0.24	0.18	0.21	0.14	0.06
ST.E.	0.18	0.18	0.19	0.19	0.20	0.20	0.21	0.21	0.21	0.21	0.21	0.22
25- 36	0.14	0.16	0.09	0.06	0.16	0.14	0.11	0.06	0.06	0.02	0.14	0.04
ST.E.	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22

MEAN DIVIDED BY ST. ERROR = 0.46936E 02

TO TEST WHETHER THIS SERIES IS WHITE NOISE, THE VALUE 0.29165E 03
SHOULD BE COMPARED WITH A CHI-SQUARE VARIABLE WITH 36 DEGREES OF FREEDOM

DIFFERENCE 1
MEAN OF THE SERIES =0.12605E 00
ST. DEV. OF SERIES =0.63480E 01
NUMBER OF OBSERVATIONS = 119

1- 12	-0.39	-0.06	-0.12	0.19	-0.14	0.00	0.07	-0.04	-0.01	-0.02	0.06	-0.08
ST.E.	0.09	0.10	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
13- 24	0.06	-0.11	0.11	0.00	0.05	-0.05	-0.00	0.03	-0.12	0.09	0.07	-0.20
ST.E.	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.12	0.12	0.12
25- 36	0.07	0.10	-0.04	-0.13	0.11	0.03	-0.01	-0.02	0.05	-0.18	0.22	-0.08
ST.E.	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.13

MEAN DIVIDED BY ST. ERROR = 0.21661E 00

TABLE E.9

PARTIAL AUTOCORRELATIONS OF AN ALTERNATING LINEAR SERIES

ORIGINAL SERIES
MEAN OF THE SERIES =0.28208E 02
ST. DEV. OF SERIES =0.65835E 01
NUMBER OF OBSERVATIONS = 120

1- 12	0.53	0.20	0.12	0.21	-0.03	0.07	0.08	-0.03	0.10	0.05	0.08	0.03
13- 24	0.10	0.04	0.16	0.07	0.04	-0.03	-0.05	-0.03	-0.08	0.03	-0.08	-0.18
25- 36	0.11	-0.03	-0.09	-0.04	0.05	-0.03	-0.04	-0.08	-0.08	-0.04	0.19	-0.12

DIFFERENCE 1
MEAN OF THE SERIES =0.12605E 00
ST. DEV. OF SERIES =0.63480E 01
NUMBER OF OBSERVATIONS = 119

1- 12	-0.39	-0.26	-0.32	-0.05	-0.18	-0.17	-0.03	-0.11	-0.07	-0.10	-0.06	-0.13
13- 24	-0.08	-0.23	-0.15	-0.10	-0.05	-0.01	-0.03	0.04	-0.11	-0.03	0.13	-0.20
25- 36	-0.06	0.03	-0.04	-0.08	-0.01	-0.00	0.08	0.08	0.08	-0.19	0.12	0.02

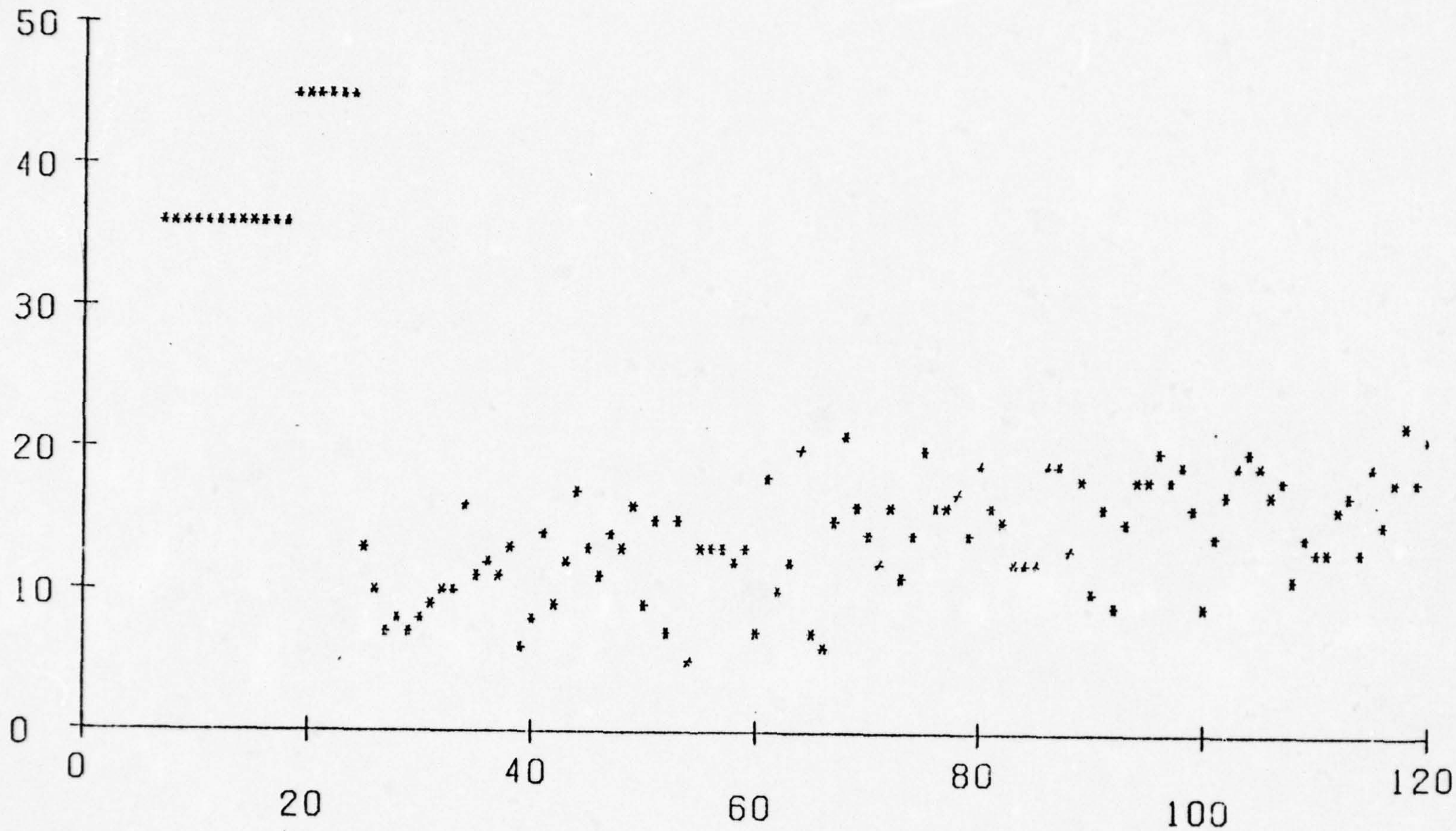


Fig. E.1. Graph of Observed Linearly Increasing Series

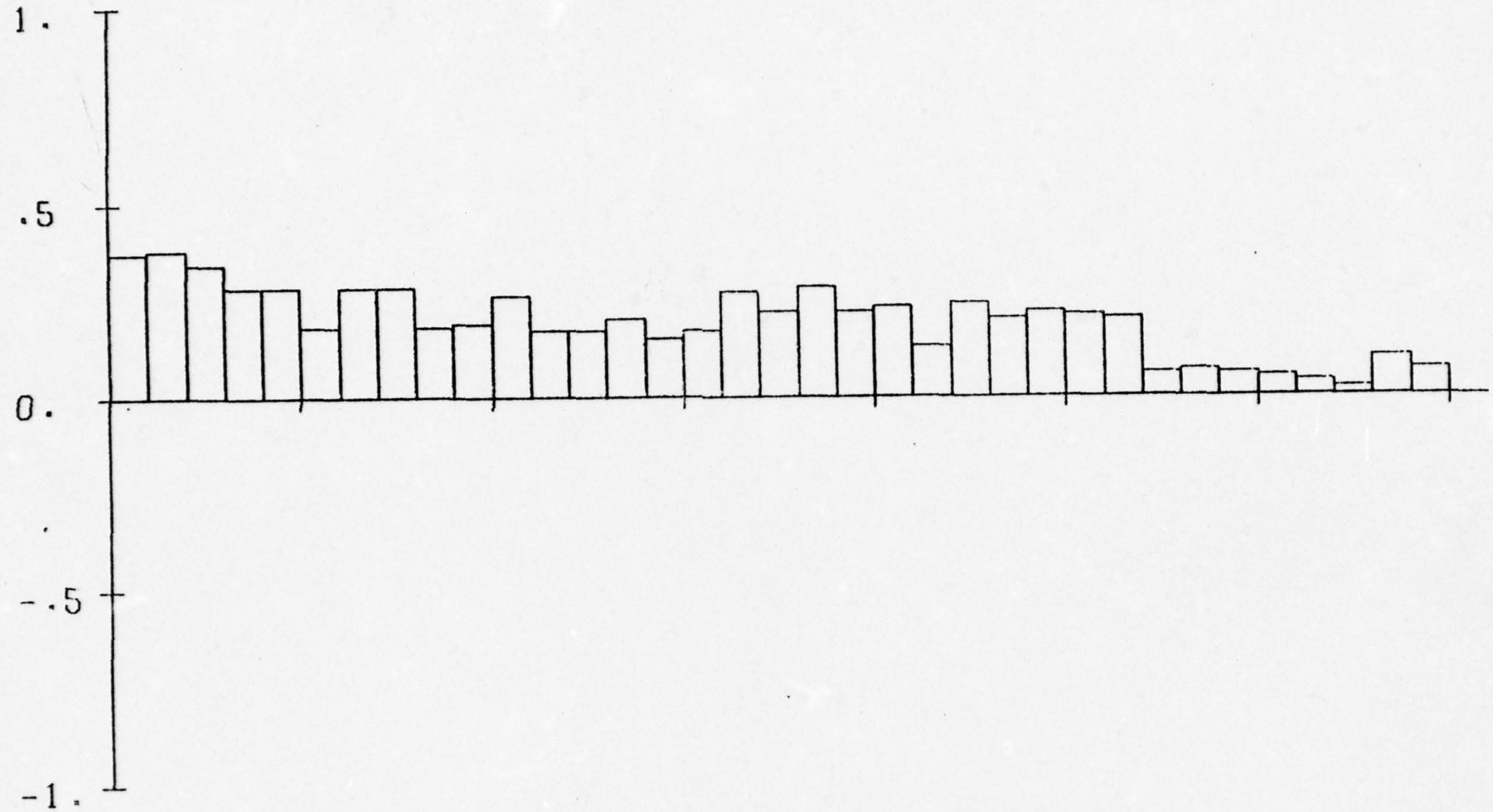


Fig. E.2. ACF of Observed Linearly Increasing Series

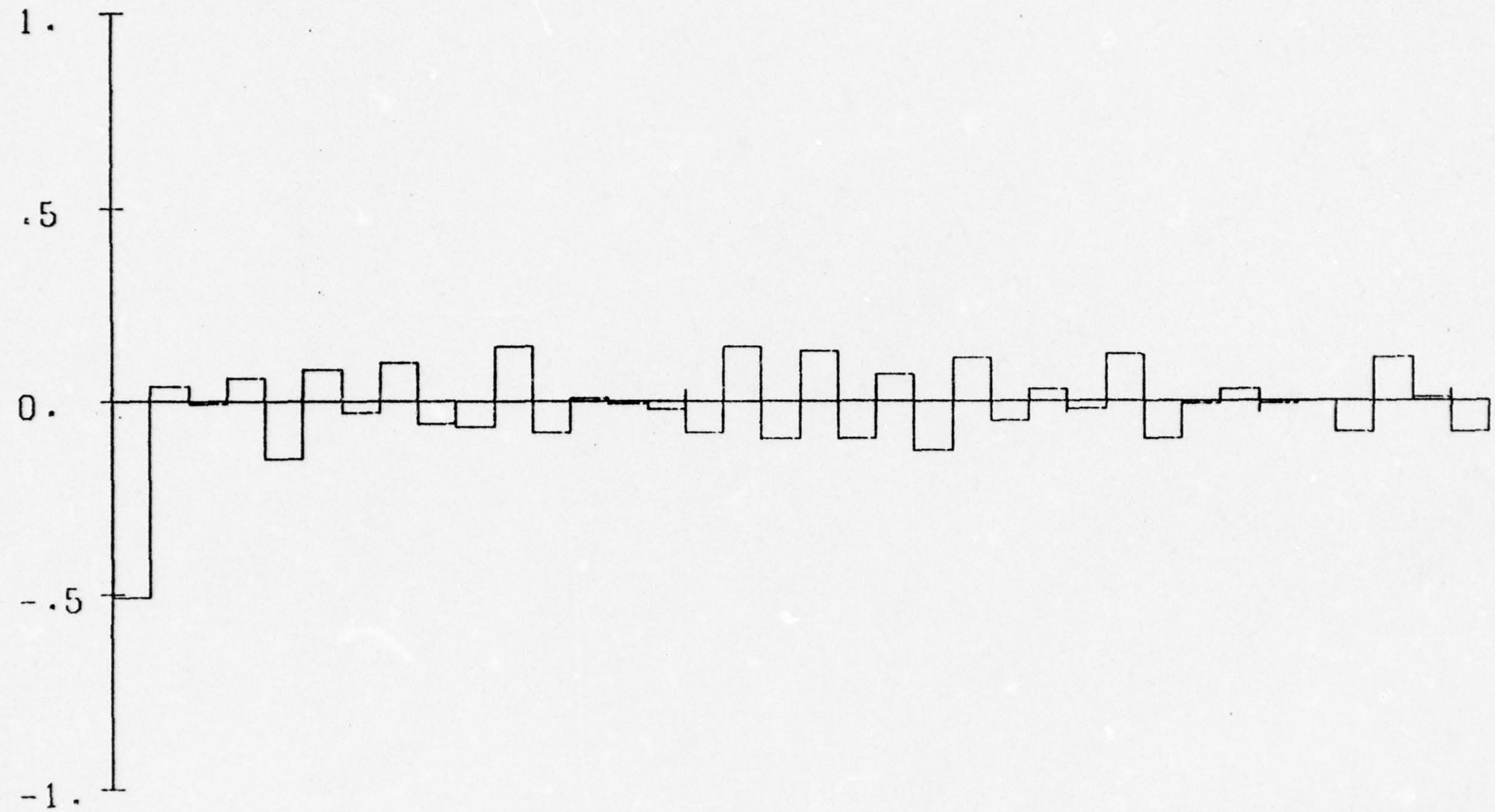


Fig. E.3. ACF of First Differences of Linearly Increasing Poisson Series

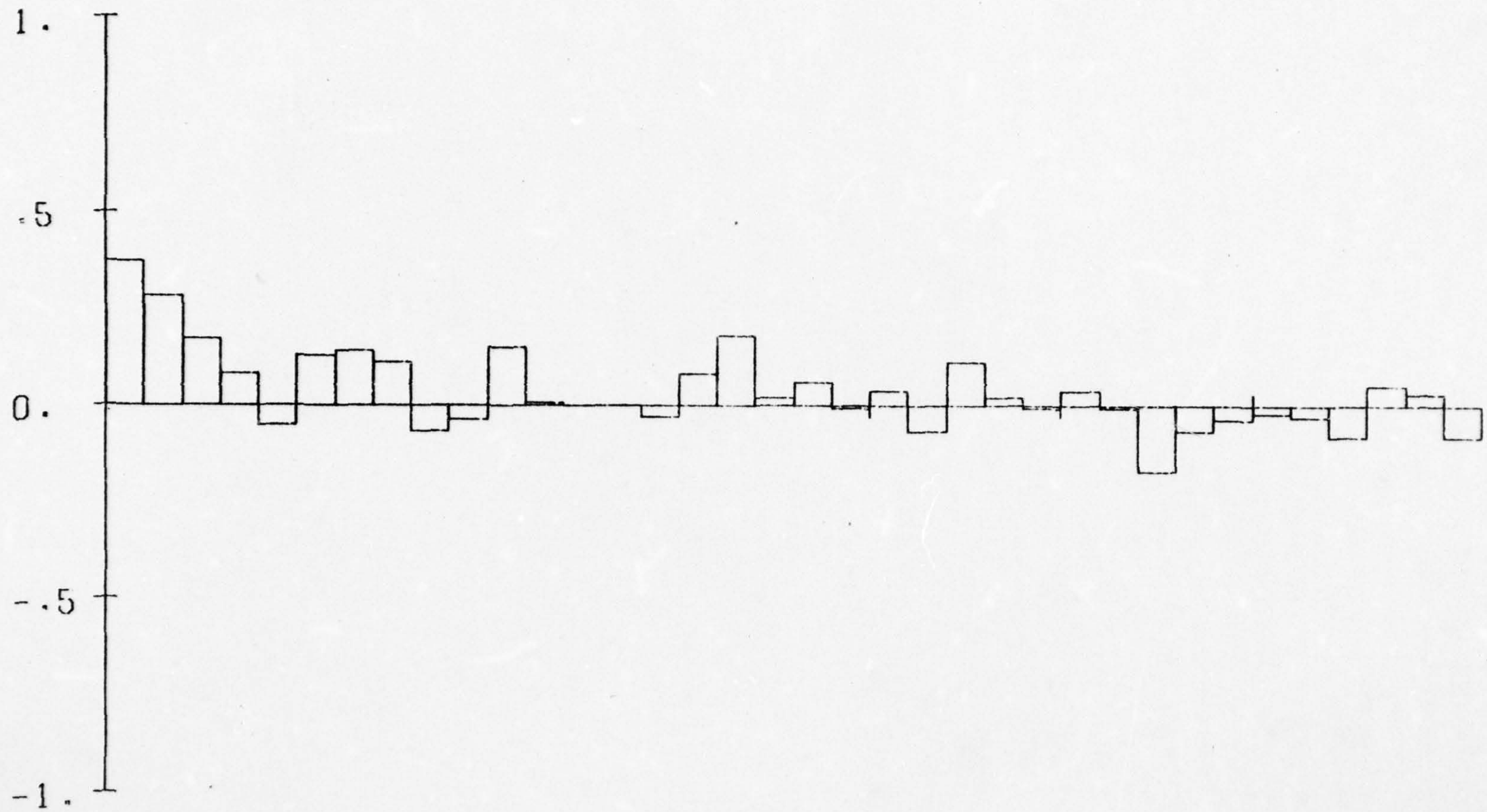


Fig. E.4. PACF of Observed Linearly Increasing Poisson Series

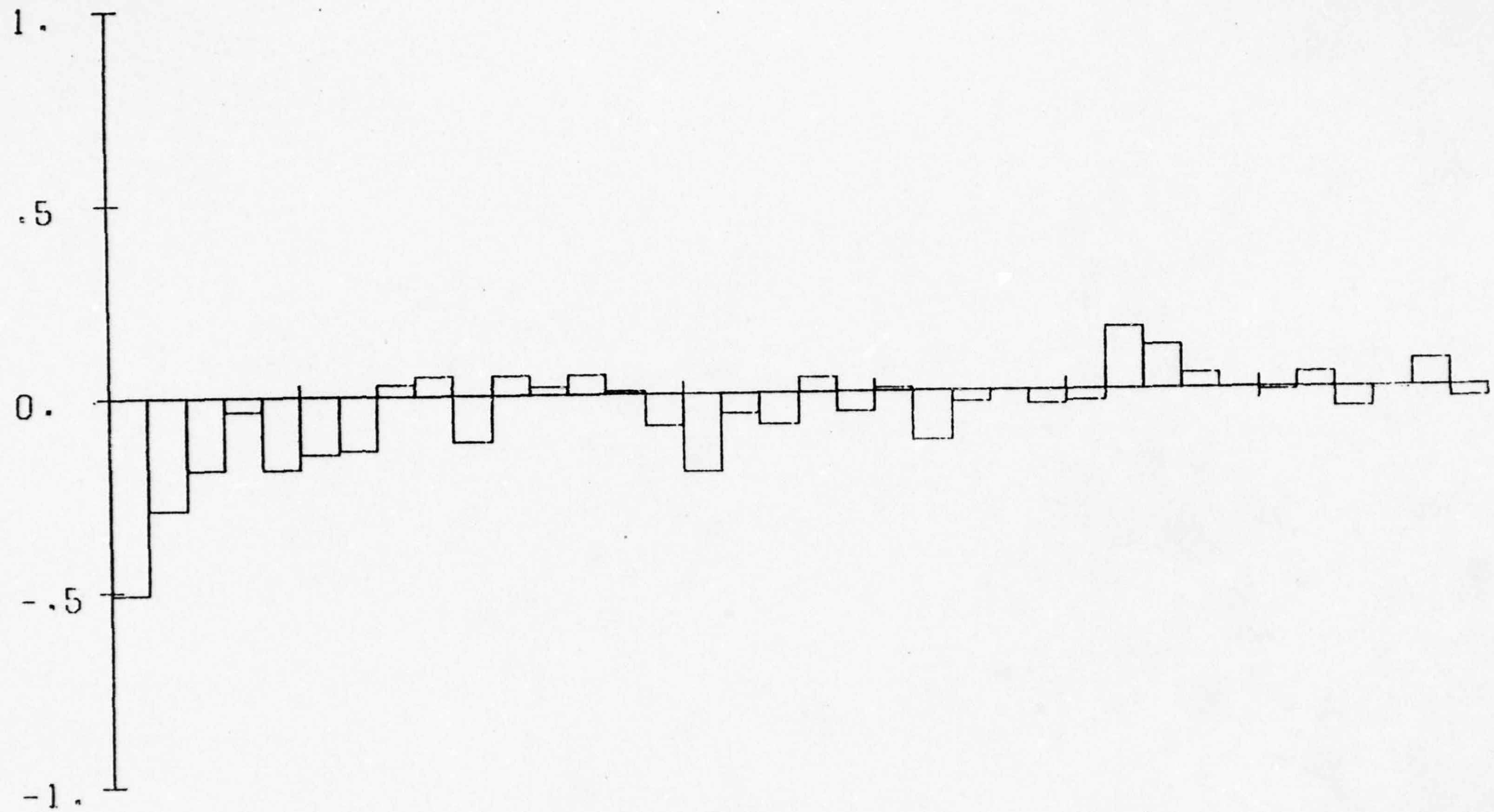


Fig. E.5. PACF of First Differences of Linearly Increasing Series

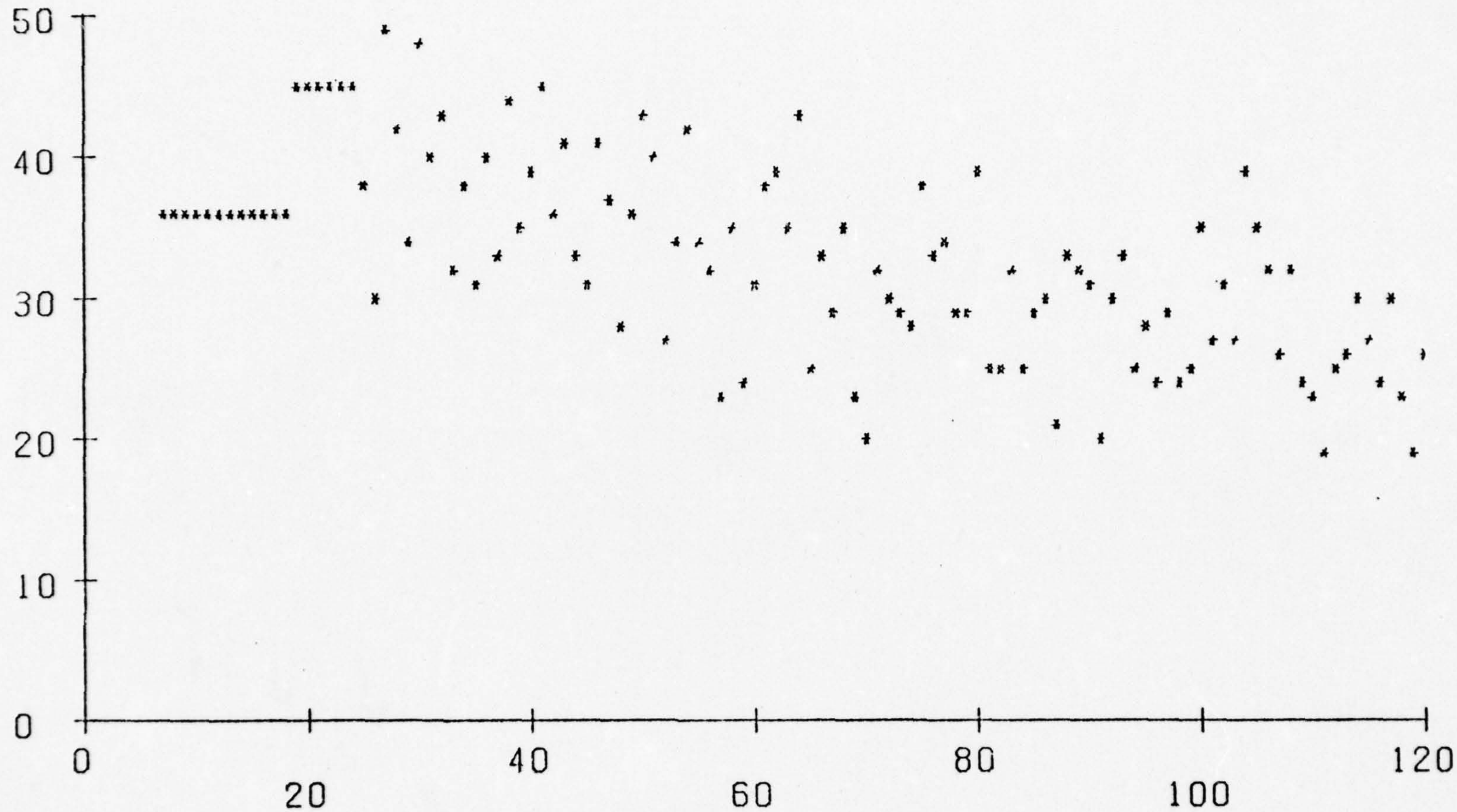


Fig. E.6. Graph of Observed Linearly Decreasing Series

Fig. E.7. ACF of Observed Linearly Decreasing Series

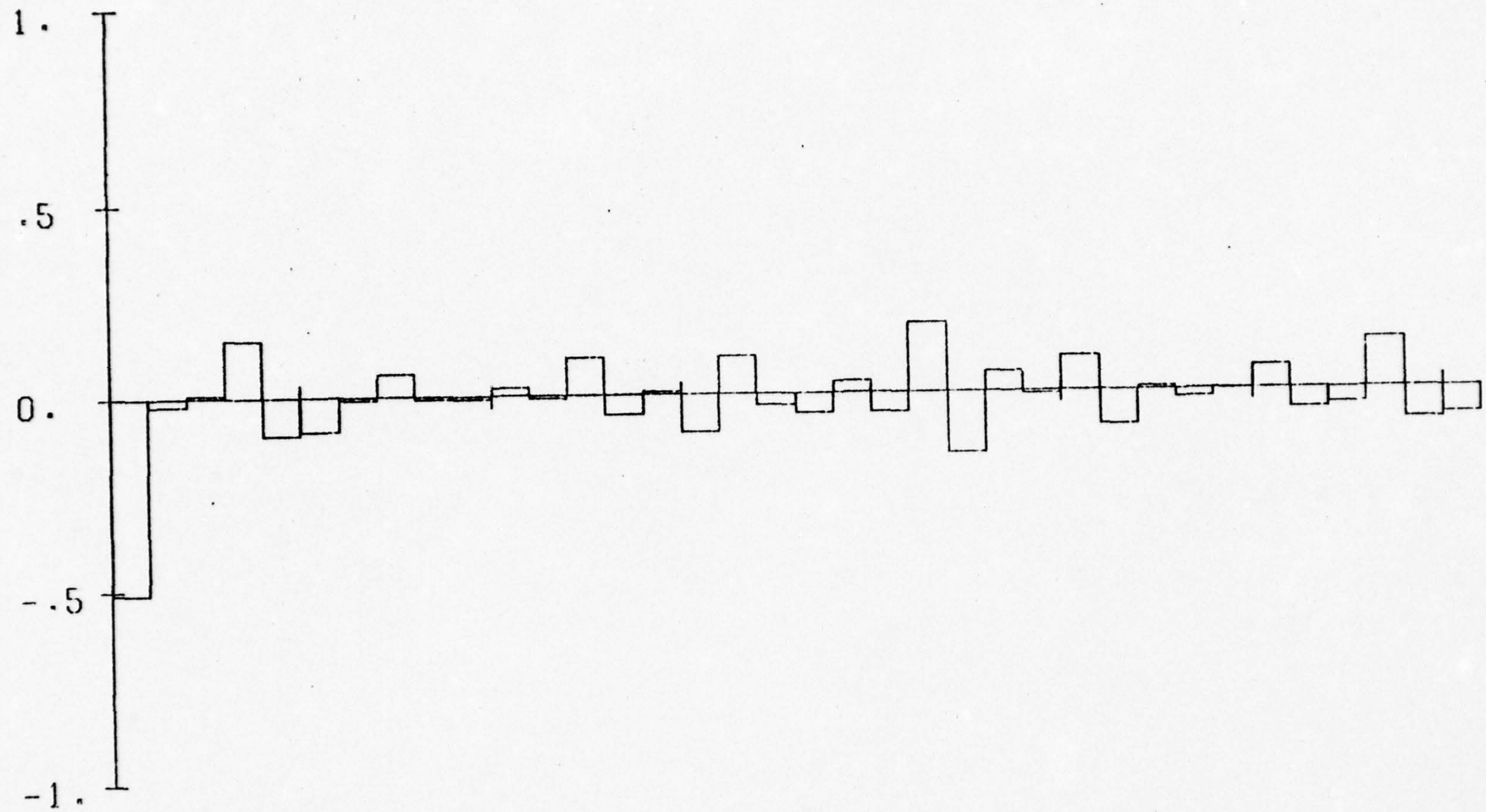


Fig. E.8. ACF of First Differences of Linearly Decreasing Series

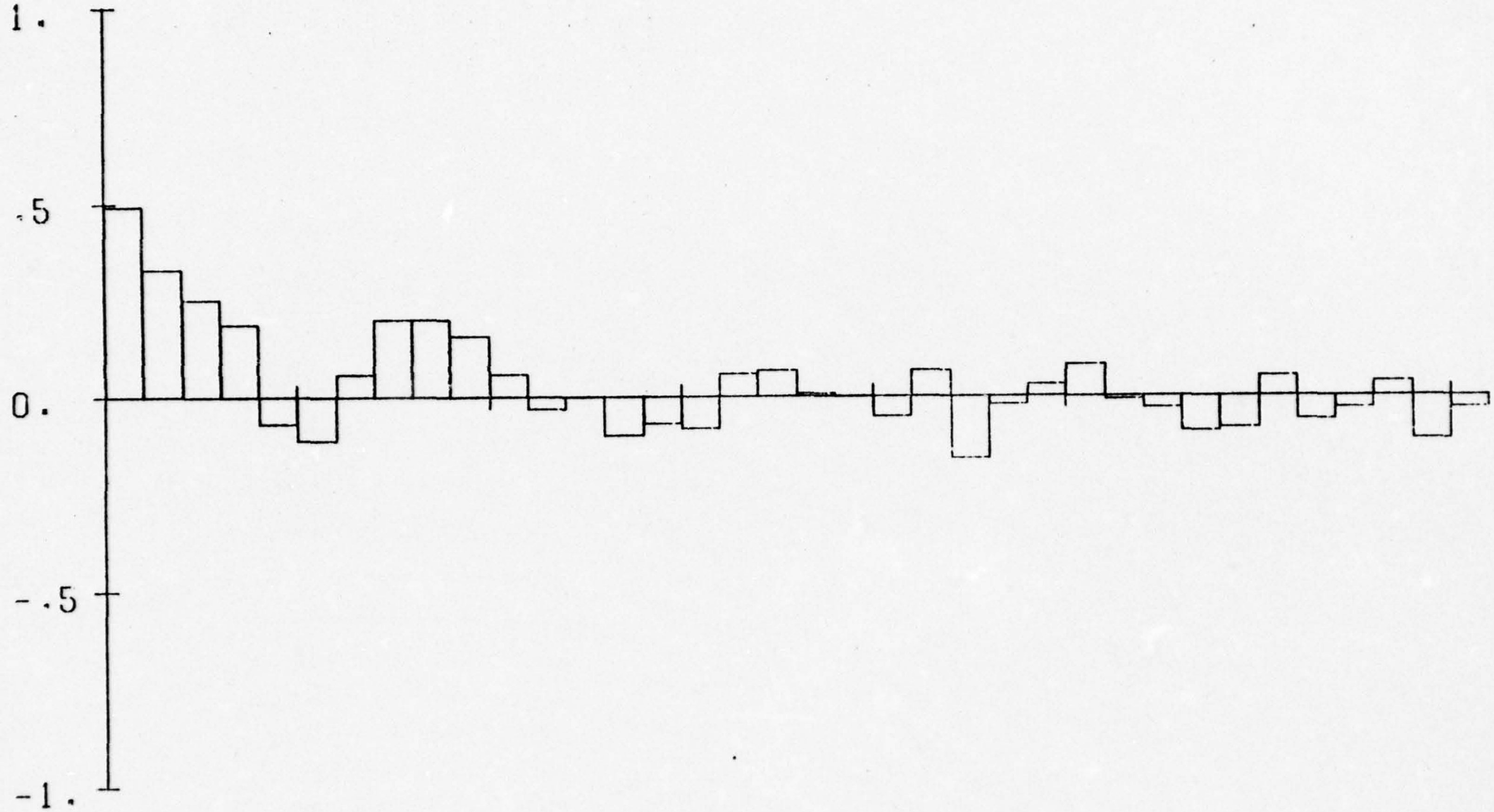


Fig. E.9. PACF of Observed Linearly Decreasing Poisson Series

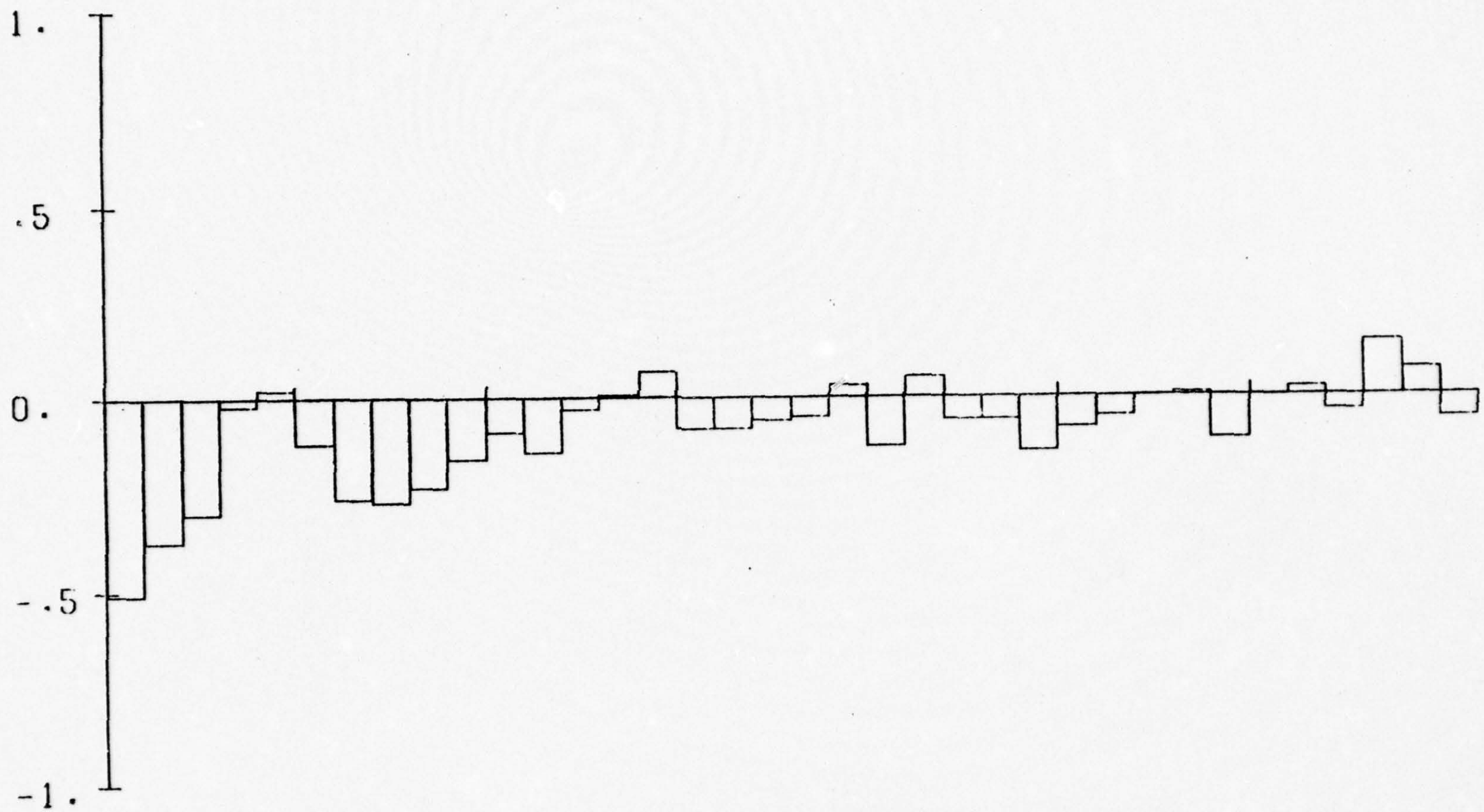


Fig. E.10. PACF of First Differences of Linearly Decreasing Series

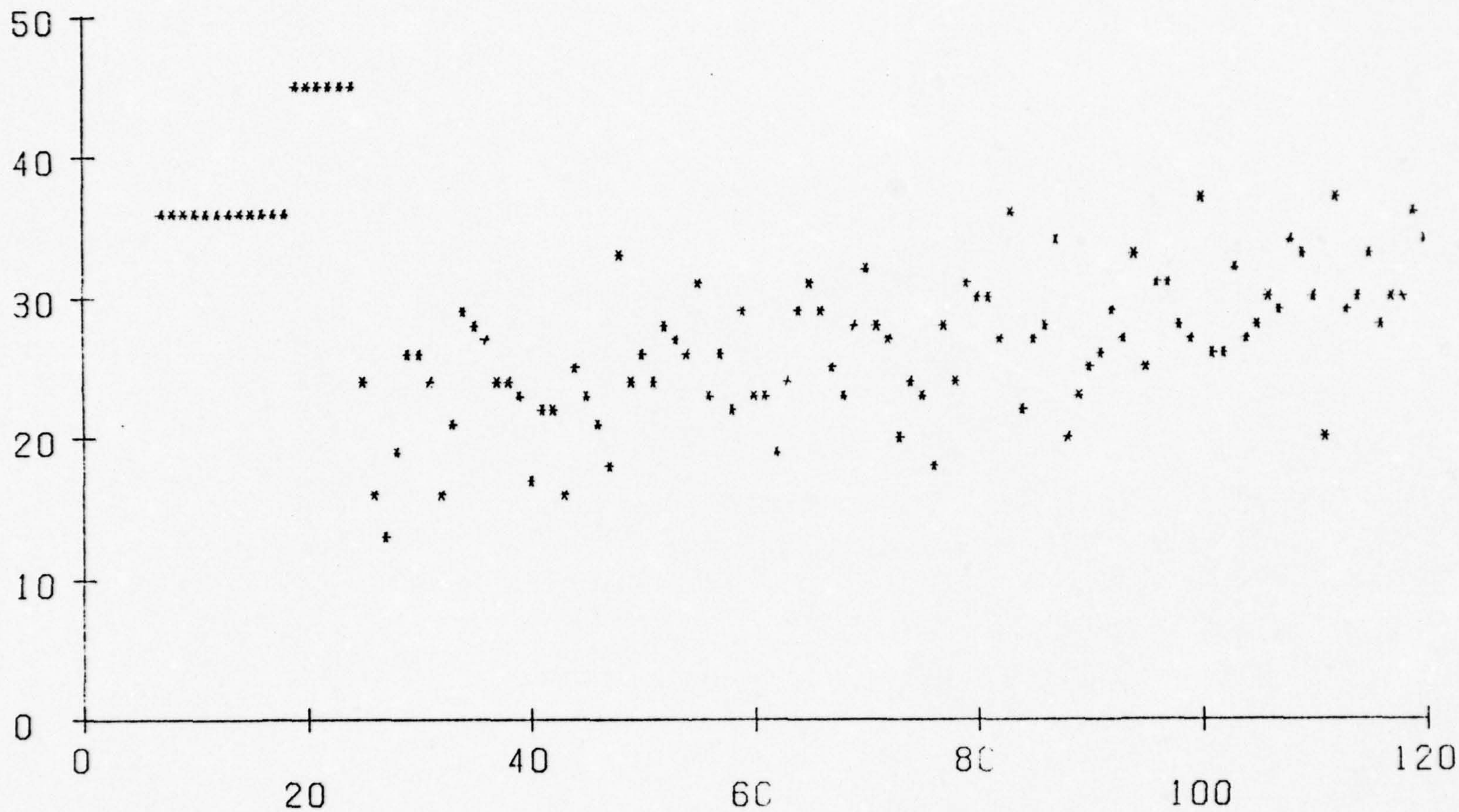


Fig. E.11. Graph of Observed Alternating Linear Series

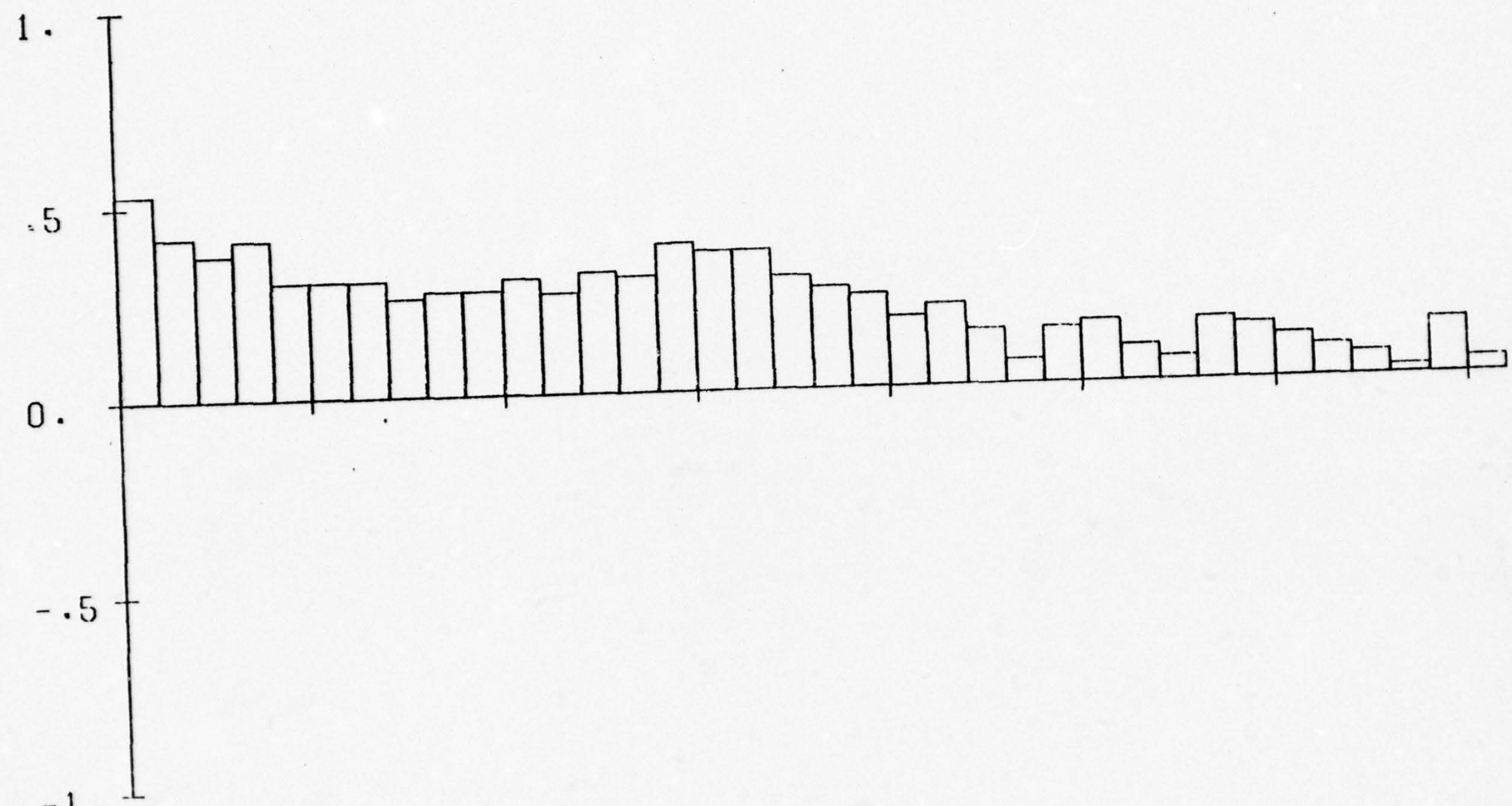


Fig. E.12. ACF of Observed Alternating Linear Series

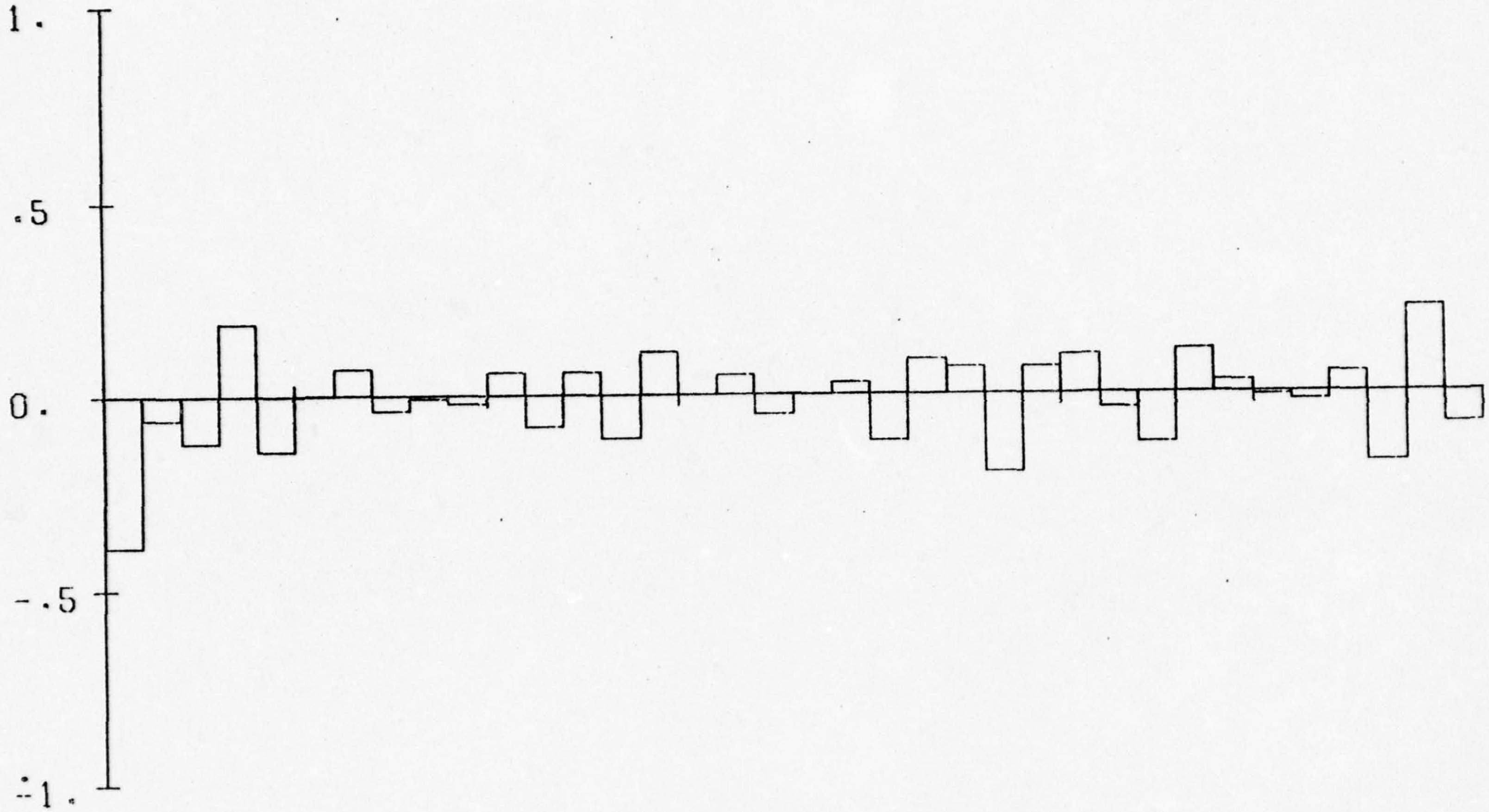


Fig. E.13. ACF of First Differences of Alternating Linear Series

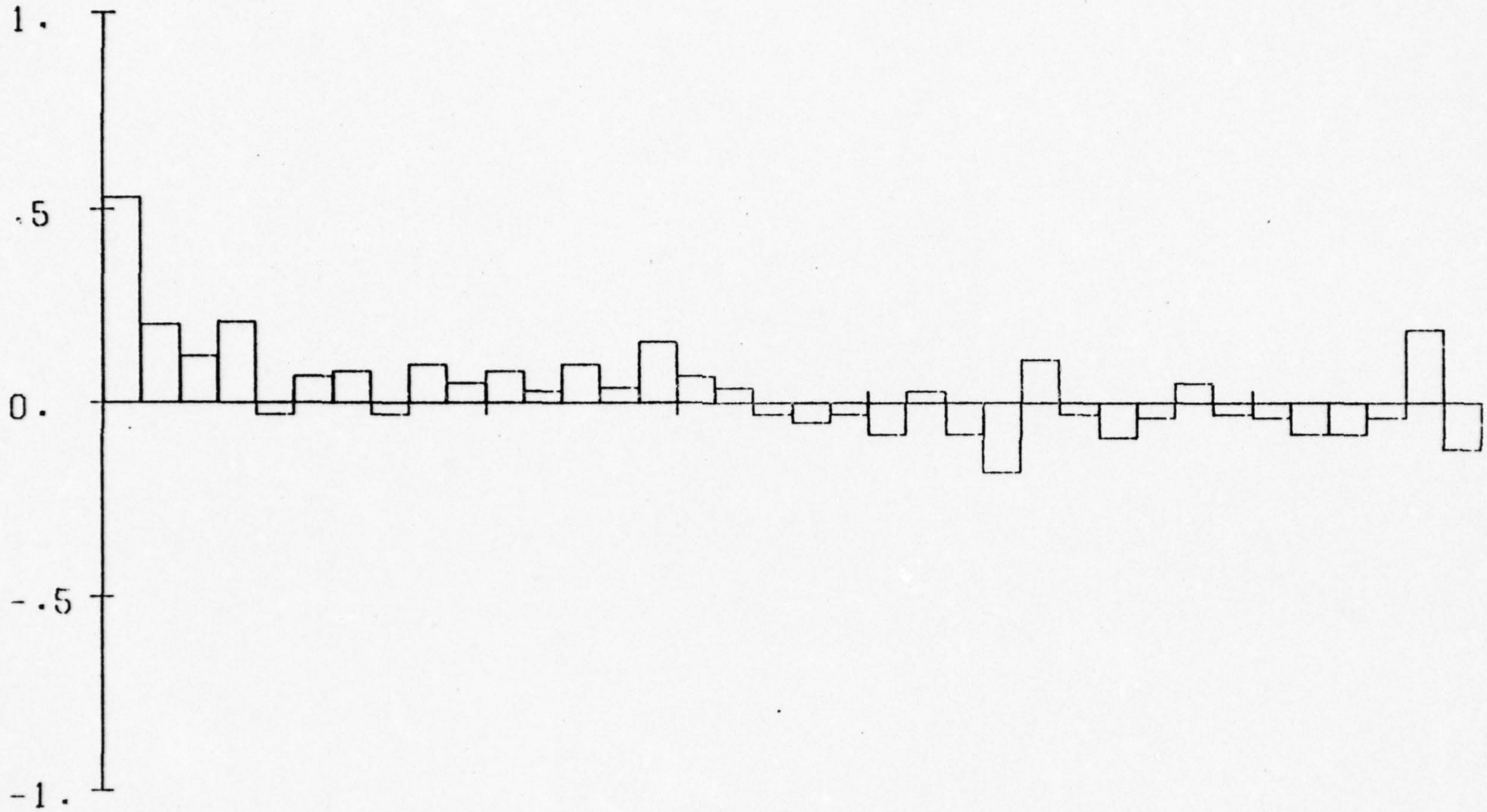


Fig. E.14. PACF of Observed Linear Poisson Series

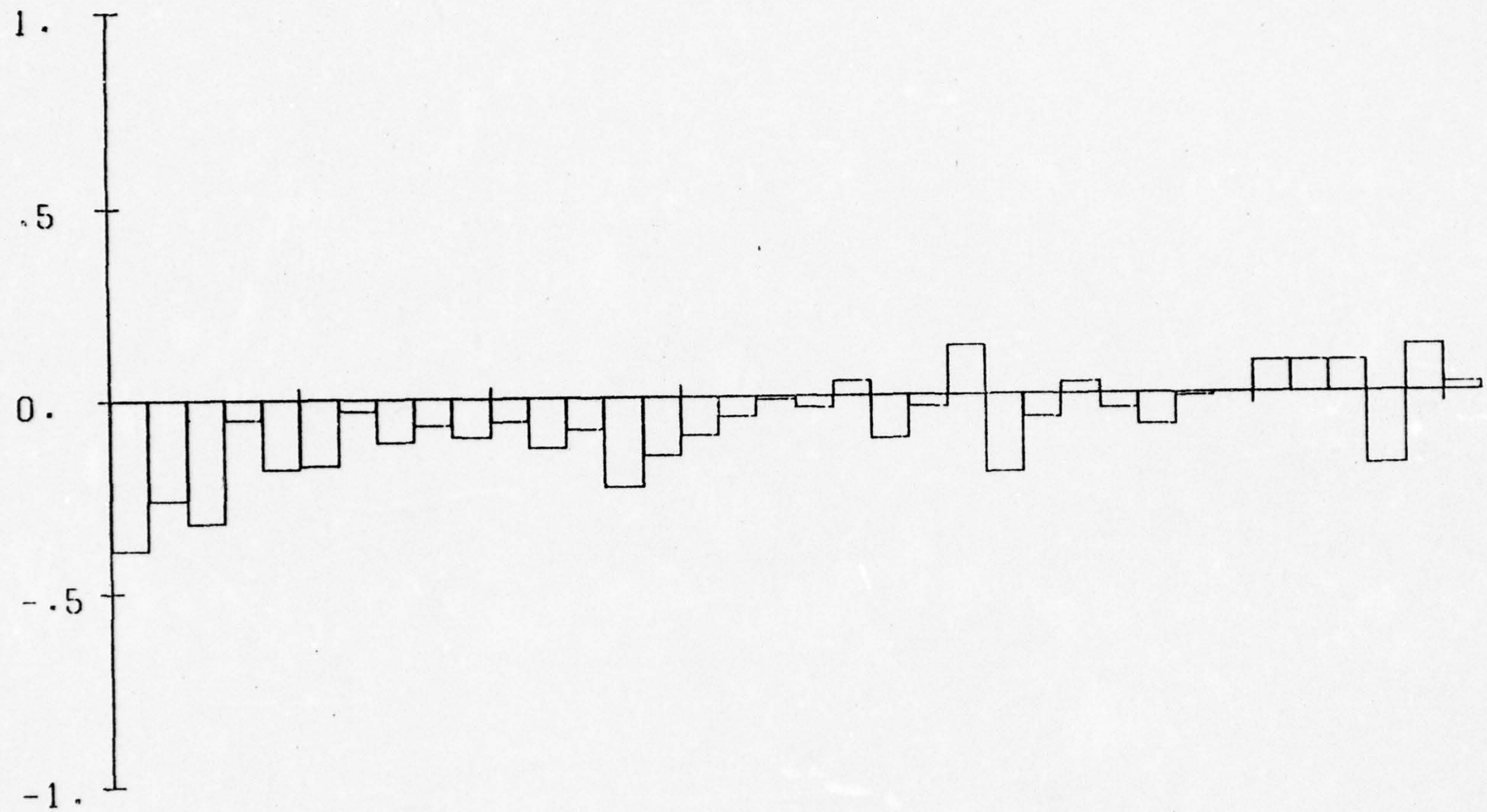


Fig. E.15. PACF of First Differences of Alternating Linear Series

Guides to Model Identification

Model identification was the most difficult time series analysis task in the research effort. Although several references on time series analysis are available, they proved to be of little value. Most of the references made little effort to fill the gap between descriptions of various models and the forecasting by the model once it had been identified. The remainder of this section will be devoted to model identification. It is a compilation of the experience gained in this research effort.

The autocorrelation and partial autocorrelation functions are very valuable tools in the identification of possible models. The Tables E.10 and E.11 and Figures E.16 to E.25 are offered as assistance in identifying possible models.

Model identification and estimation overlap (1:173). Employment of estimation procedures was used to carry out part of the identification. Identification is necessarily somewhat inexact since a model is being built to fit the data. It is in this stage that graphical methods and judgement coming from experience are particularly useful. It should be noted that preliminary identification does nothing but tentatively identify a class of models which can later be more closely fitted to the data and checked for consistency.

TABLE E.10

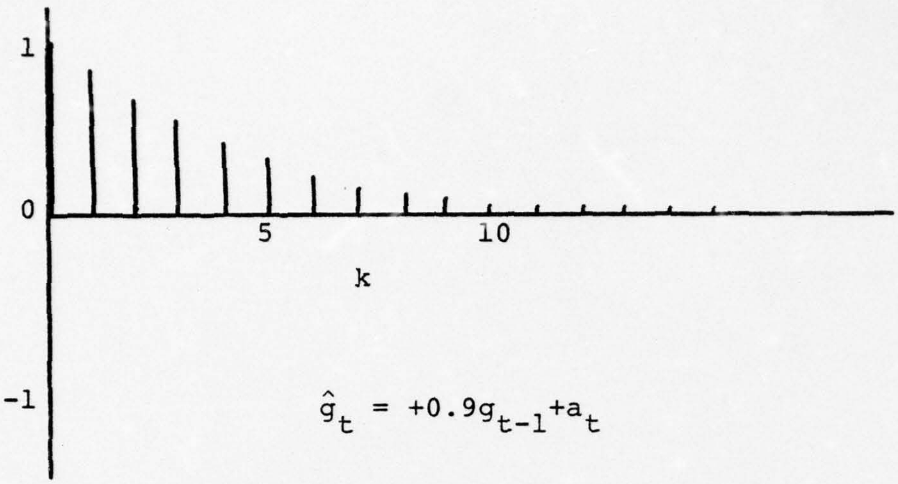
BEHAVIOR OF AUTOCORRELATION FUNCTIONS WITH PARAMETER ρ_k

Model	Behavior of ρ_k
AR (1,d,0)	Decays exponentially (see Figure E.16)
MA (0,d,1)	Only ρ_1 is nonzero (see Figure E.17)
AR (2,d,0)	Mixture of exponentials of damped sine wave (see Figure E. 18)
MA (0,d,2)	Only ρ_1 and ρ_2 are nonzero (Figure E.19)
ARIMA(1,d,1)	Decays exponentially from first lag (see Figure E.20)

TABLE E.11

BEHAVIOR OF AUTOCORRELATION FUNCTIONS WITH PARAMETER ϕ_{kk}

Model	Behavior of ϕ_{kk}
AR (1,d,0)	Only ϕ_{11} is nonzero (see Figure E.21)
MA (0,d,1)	Exponential decay (tail off) (see Figure E.22)
AR (2,d,0)	Only ϕ_{11} and ϕ_{22} are nonzero (see Figure E.25)
MA (0,d,2)	Designated by mixture of exponentials or damped sine wave (see Figure E.24)
ARIMA(1,d,1)	Dominated by exponential decay from first lag (ϕ_{11}) (see Figure E.25)



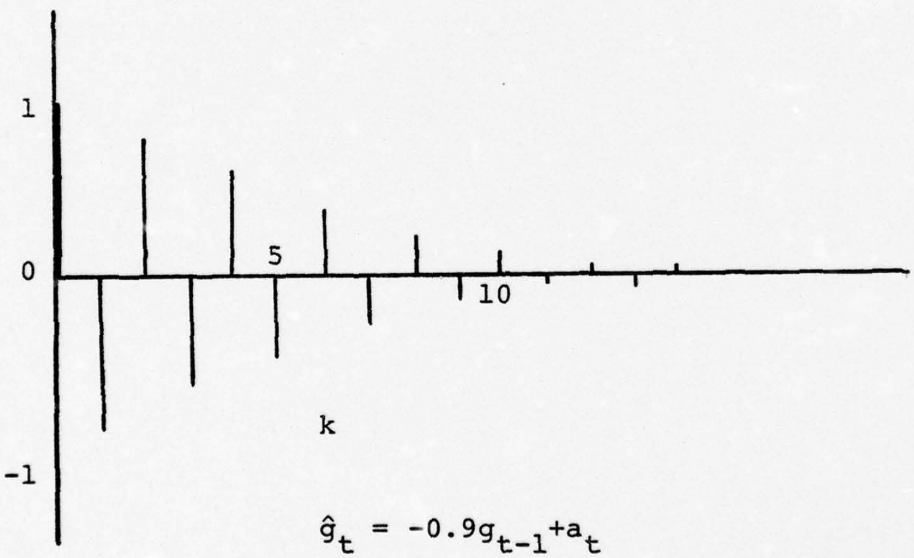


Fig. E.16. Autocorrelograms of A AR (1,d,0) Process

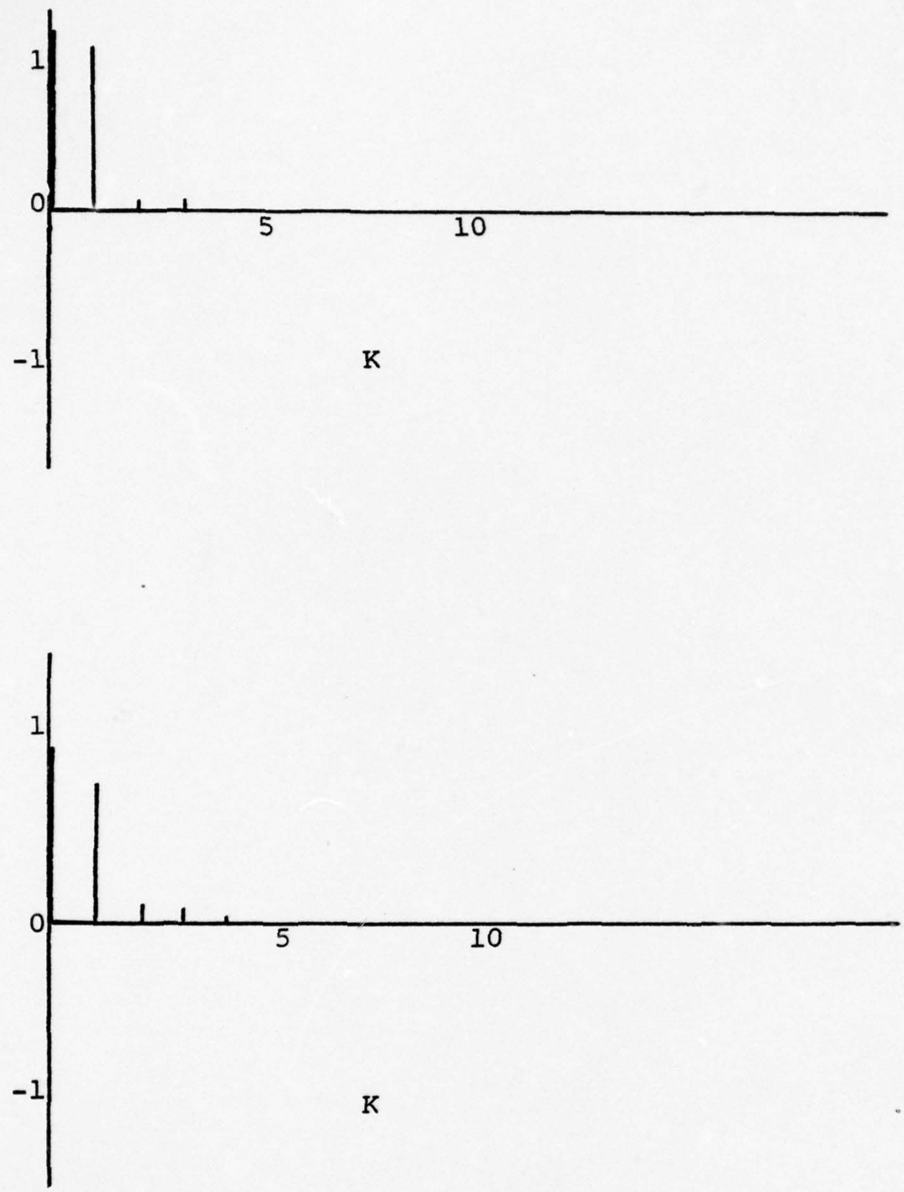


Fig. E.17. Autocorrelograms of A MA (0,d,1) Process

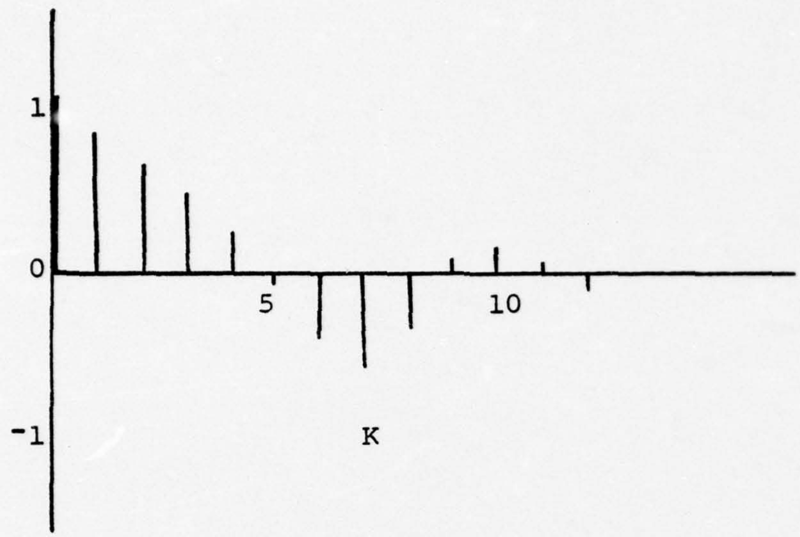


Fig. E.18. Autocorrelogram of A AR (2,d,0) Process

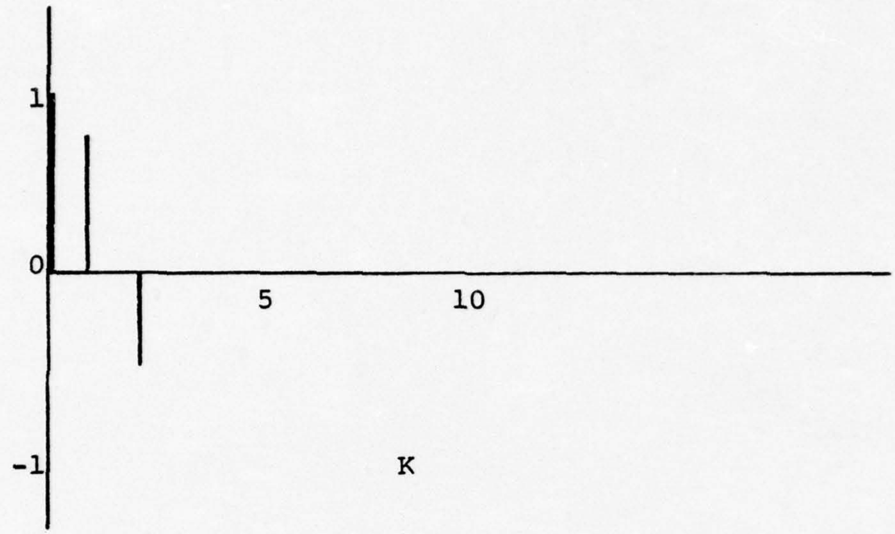


Fig. E.19. Autocorrelogram of A MA (0,d,2) Process

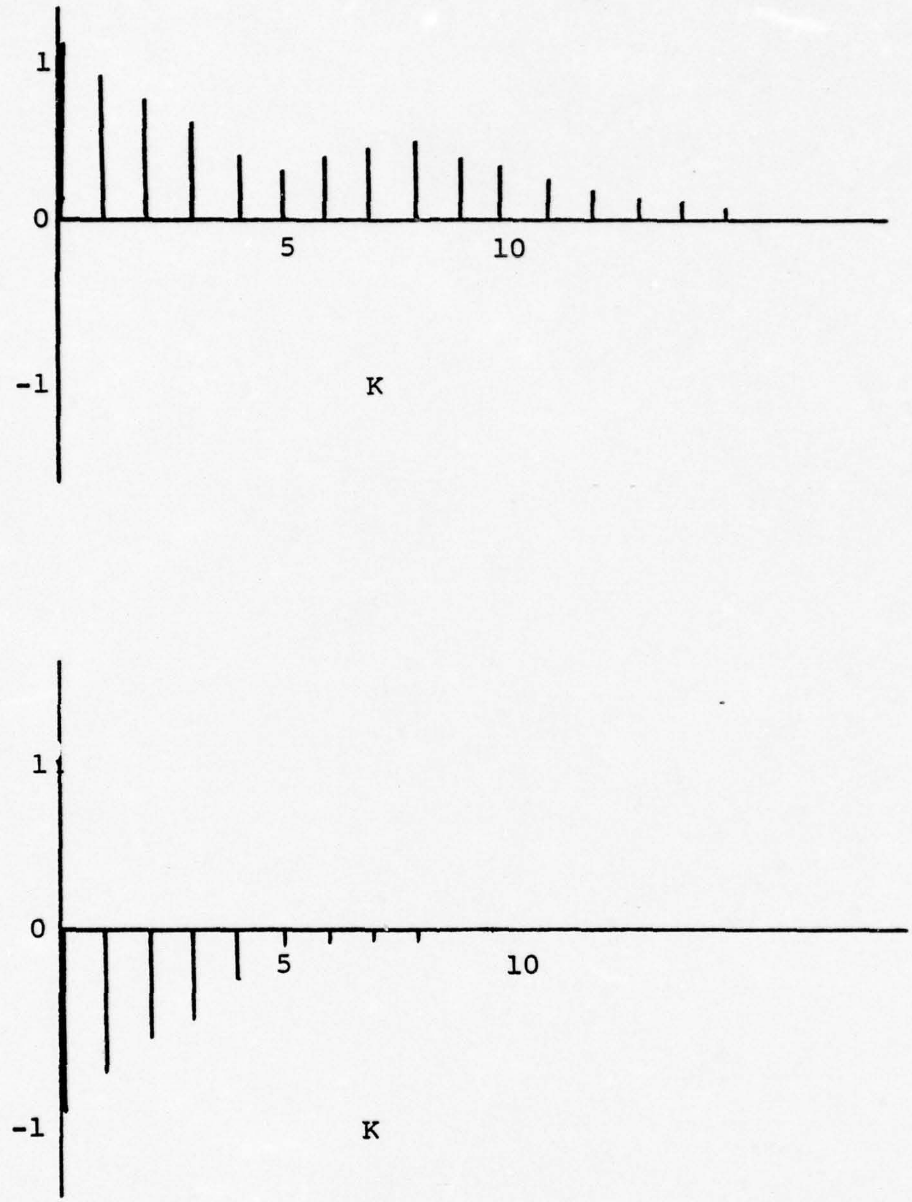


Fig. E.20. Autocorrelograms of A ARIMA (1,d,1) Process

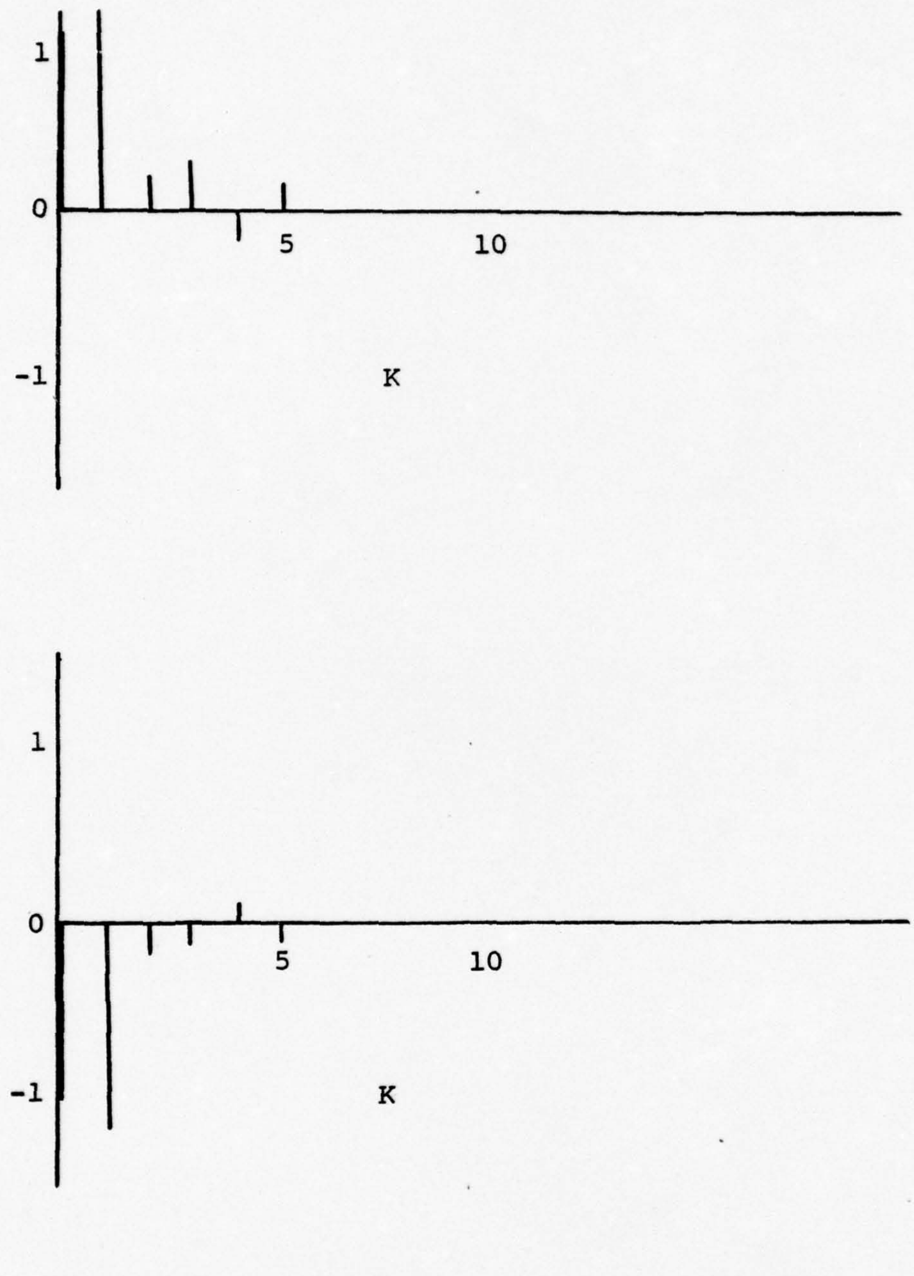
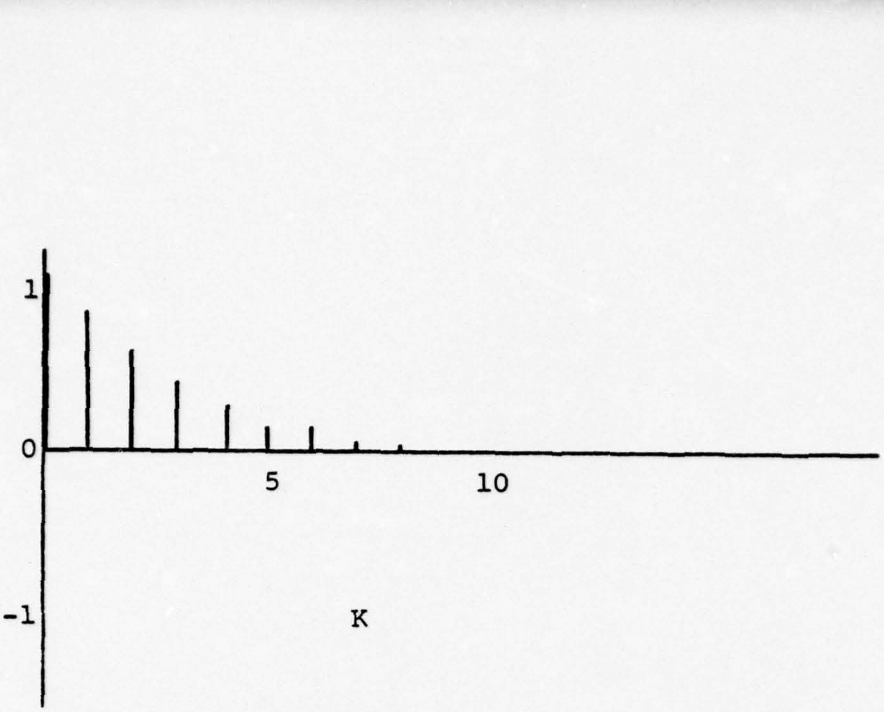


Fig. E.21. Partial Autocorrelogram of A AR (1,d,0) Process



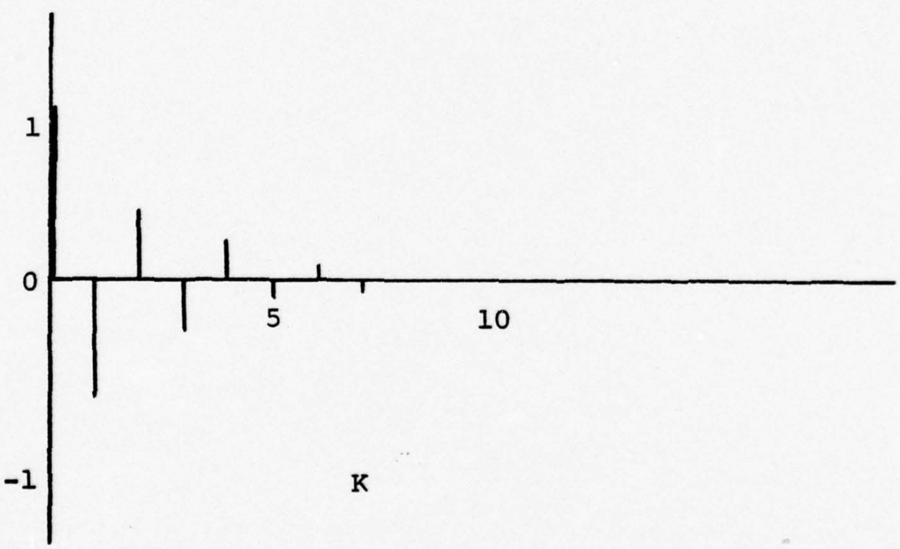


Fig. E.22. Partial Autocorrelogram of A MA (0,d,1) Process

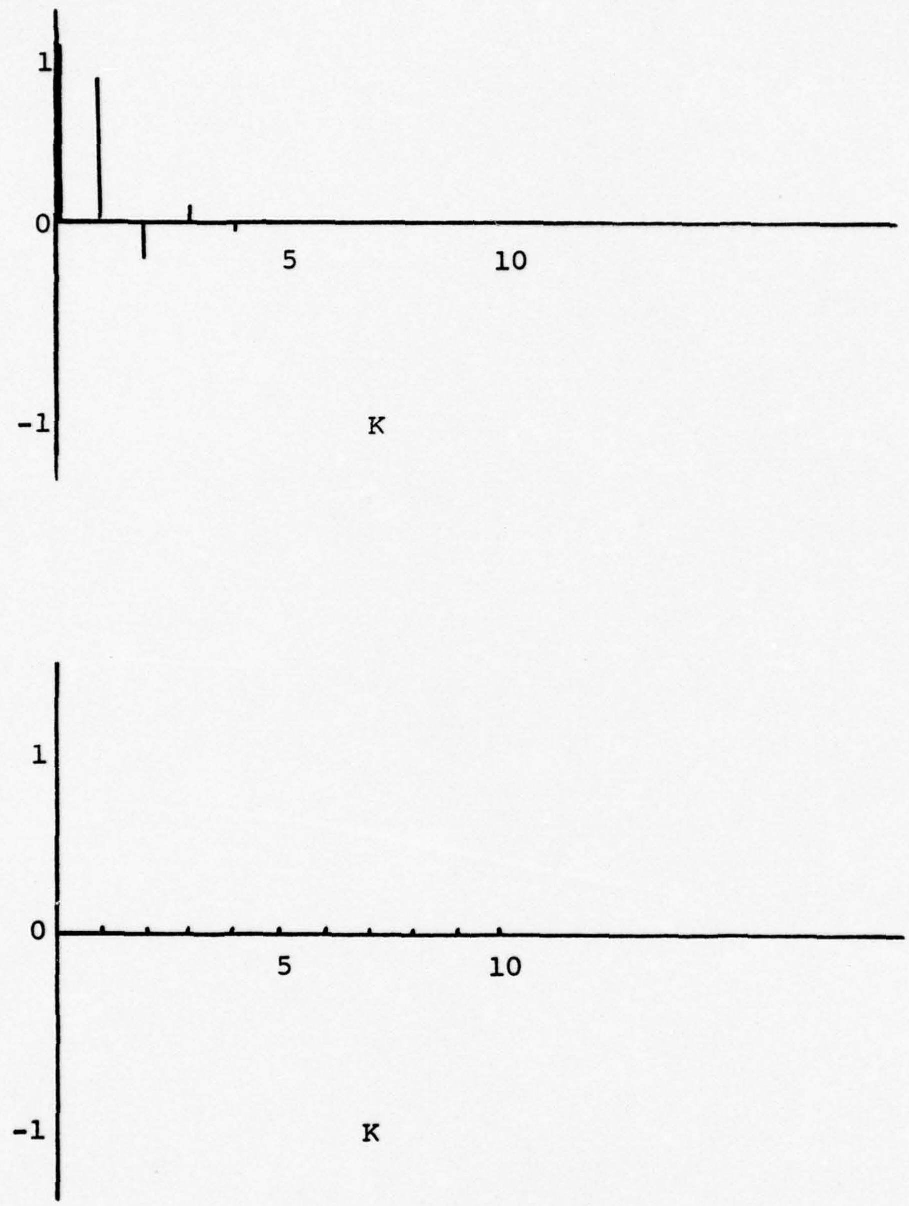


Fig. E.23. Partial Autocorrelogram of A AR (2,d,0) Process

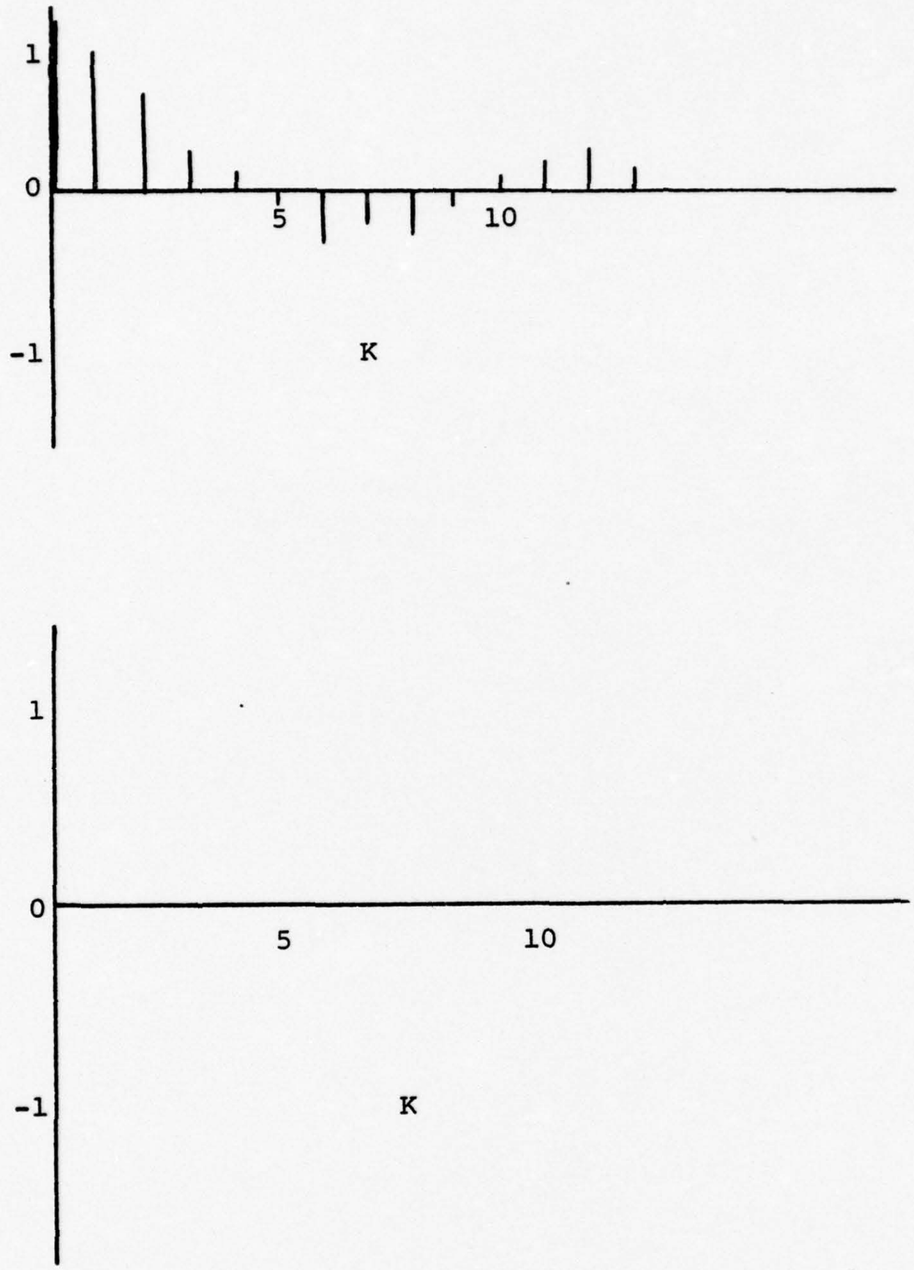


Fig. E.24. Partial Autocorrelogram of A MA (0,d,2) Process

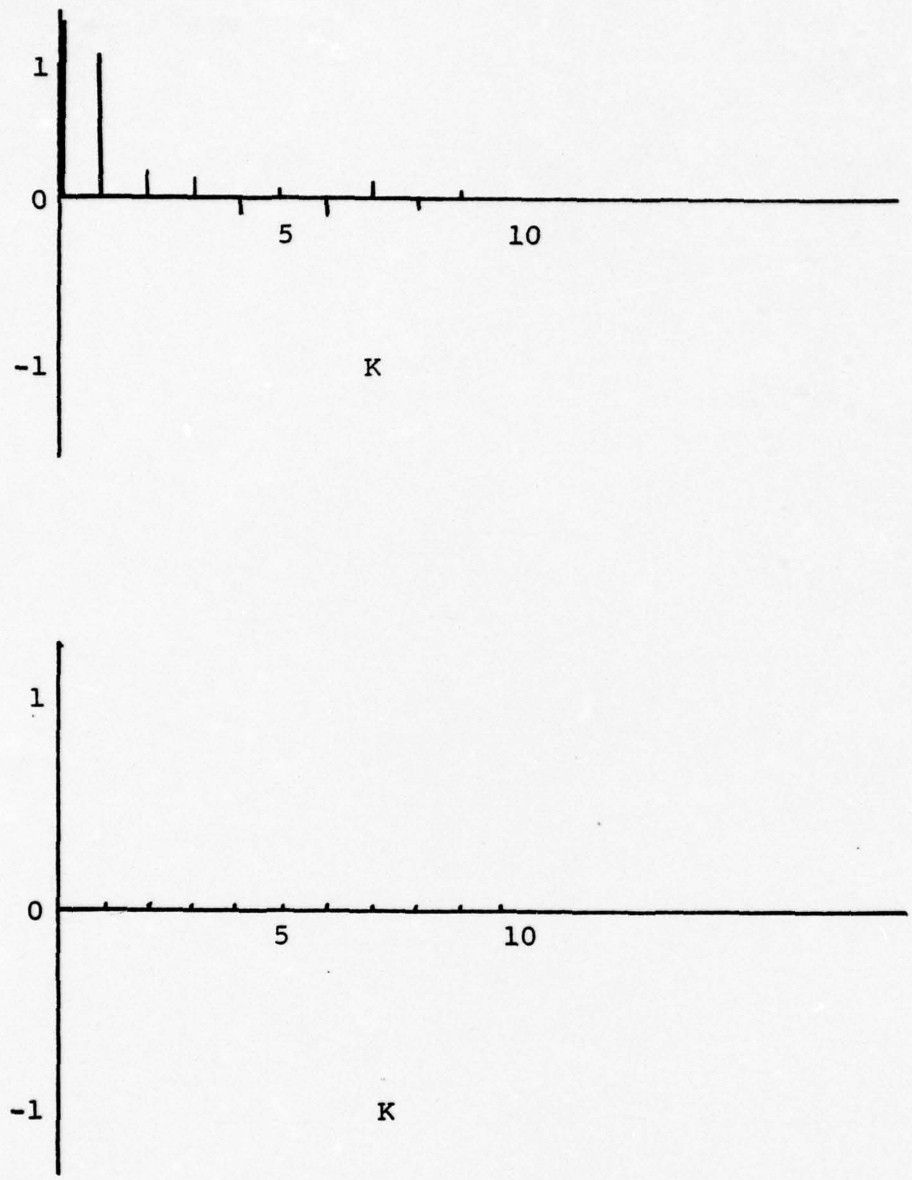


Fig. E.25. Partial Autocorrelogram of A ARIMA (1,d,1) Process

The estimated autocorrelation function of the increasing linear Poisson process (see Figure E.2) tends to die out quickly, and this indicates that the degree of differencing is zero (i.e., d=0). Since both the autocorrelation function and partial autocorrelation function had an exponential appearance the partial autocorrelation function tailed off (see Figures E.2 and E.4), a mixed autoregressive-moving average process of the form ARIMA (1,0,1) was selected as the model. Similarly, in the examination of the correlegrams for the linearly decreasing and alternating linear Poisson patterns (see Figures E.7, E.9, E.12 and E.14), the same exponential appearance with dampened sine patterns was observed in the autocorrelation function with a cutoff of the partial autocorrelation function after the first lag. The ARIMA (1,0,1) form was again suggested.

Once a model has been tentatively selected, the next step is to estimate the parameters.

Estimation of Parameters

Initial Estimates for Autoregressive Processes.

For an assumed AR process of order 1 or 2, initial estimates for the parameters ϕ_1 and ϕ_2 can be calculated by substituting estimates r_b for the theoretical autocorrelations ρ_b. For an AR(1):

$$\hat{\phi}_{11} = r_1$$

where r_1 is the first autocorrelation value in the output from UNIDEN. For an AR(2):

$$\phi_{21} = \frac{r_1(1-r_2)}{1-r_1^2} \quad \text{and} \quad \phi_{22} = \frac{r_2-r_1^2}{1-r_1^2} \tag{E.1}$$

where ϕ_{pb} denotes the estimate of the b^{th} autoregressive parameter in an autoregressive process of order p, and r_2 is the second autocorrelation value in the output from UNIDEN (see Tables E.2, E.5, E.8). (The above results are obtained from the Yule-Walker equations.)

Initial Estimates for Moving Averages. The general equation for moving average is:

$$r_k = \frac{-\hat{\phi}_k+\hat{\phi}_1\hat{\phi}_{k+1}+\hat{\phi}_2\hat{\phi}_{k+2}+\ldots+\hat{\phi}_{q-k}\hat{\phi}_q}{(1+\hat{\phi}_1^2+\hat{\phi}_2^2+\ldots+\hat{\phi}_q^2)}. \tag{E.2}$$

The reader is referred to Table D, page 517, in the Box and Jenkins text to obtain preliminary estimates for a moving average process of q=1. ρ_1 will be related to θ_1 and by substituting $r_1(w)$ for ρ_1, the Table can be used to provide initial estimates for an (0,d,1) process. Similarly, Chart C, page 519, relates ρ_1, and ρ_2, to

θ_1 and θ_2, by substituting $r_1(w)$ and $r_2(w)$ for ρ_1, and ρ_2, initial estimates for a (0,d,2) process can be obtained.

<u>Initial Estimates for a Mixed Autoregressive Average Process</u>. For order p=1,d, q=1, estimates can be obtained by substituting $r_1(w)$ and $r_2(w)$ for ρ_1, and ρ_2 in the following equation:

$$r_1 = \frac{(1-\hat{\theta}_1\hat{\phi}_1)(\hat{\phi}_1-\hat{\theta}_1)}{1+\hat{\theta}_1^2 - 2\hat{\phi}_1\hat{\theta}_1} \qquad \text{(E.3)}$$

$$r_2 = r_1\phi_1 \ .$$

These equations can easily be solved, since r_1 and r_2 are the first and second autocorrelation values found in UNIDEN (see Tables E.2, E.5, and E.8). Chart D (1:520) in the Box and Jenkins text relates ρ_1 and ρ_2 to ϕ_1 and θ_1 which can be used to provide initial estimates of the parameters for any ARIMA (1,d,1) process. For example, the estimates for a linearly increasing series were computed to be: r_1=37, r_2=.28 $\hat{\phi}_1$=.76, and $\hat{\theta}_1$=.48. The program provided estimates of the mean and variance of the series to be $\hat{\mu}$=14.7 and $\hat{\sigma}^2$=20.25 (see Table E.2)

Other examples are:

Linearly Decreasing Poisson Pattern (see Table E.5)

r_1=.49, r_2=.49, $\hat{\phi}_1$=1, $\hat{\theta}_1$=1, $\hat{\mu}$=30.05, and $\hat{\sigma}^2$=51.84.

Alternating Linear Poisson Pattern (see Table E.8)

r_1=.53, r_2=.42, $\hat{\phi}_1$=.79, $\hat{\theta}_1$=.30, $\hat{\mu}$=.28.20, and $\hat{\sigma}^2$=43.30.

These estimates and parameters for the models were built into a forecasting file using the file building computer program in Appendix C.

The Use of AFIT.LIB/UNEST,R for Parameter Estimation and Forecasting

The dimension limitations of AFIT.LIB/UNEST,R, hereafter referred to as UNEST, are:

25 Parameters

150 Autocorrelations of residuals

150 Partial autocorrelations of residuals

300 Forecasts

5 Time origins for forecasts

25 New observations to use in calculating update forecasts

500 Observations in series

800 Total observations and forecasts

200 Maximum backorders on either side of model

The variables in the program which are user supplied include those used in UNIDEN. The following list includes only new variables or those with a different use in UNEST:

EPSI The maximum change in the relative sum of squares before iteration in calculation of parameter stops. Set = .00 to suppress function.

ESP2 The maximum relative change in each parameter before iteration stops. Again it can be suppressed by setting it equal to .00.

ICI The control on the width of the confidence limits for the forecasts. The values of 1, 2, 3, 4, 5 relate to 50, 75, 90, 95, 99 percent limits respectively.

ILDEST The listing function of the program for the estimation routines. It can be suppressed by entering 0 for ILDEST.

ILDFCA The listing of data by forecasting routine. It can be suppressed by setting equal to 0.

INC The array containing MFAC(1)+MFAC(3)+2 numbers of each of the specified types of parameters in the model to be used.

IOPA An array containing the order of each parameter from left to right in the time series model being used (these are the powers of the B operators).

IPDEST The plotting of data estimation routine. It can be surpressed by setting equal to zero.

IPDFCA The plotting of the data from the forecasting routine. Set = 0 to suppress.

IPRES The plotting of residuals routine. It can be suppressed by setting equal to zero.

IWTPA The plotting of the residual autocorrelations. It can be suppressed by setting equal to zero.

MCSE The function which calculates the standard errors of the residual autocorrelation. It can be suppressed by setting equal to zero.

MFAC An array of size 3 where the orders of the ARIMA (p,d,q) model are enterd. MFAC(1) is the number of autoregressive factors; MFAC(2) is the number of differencing factors; MFAC(3) is the number of moving average factors in the time series model.

MIT The maximum number of iterations in the estimation routine. The maximum that will be accepted by the program is 999.

NF The number of forecasts.

NT The array containing the forecast time origins.

NTO The number of time origins to use in forecasting. The forecasts (NF) are made from

each of the time origins. Warning: *some graph titles may not turn out if NTO exceeds 9.*

NU The number of new observations to be used in updating the forecasts.

PA The array where the initial estimates of the parameters are stored for the estimation routines.

The use of CAFILE will greatly simplify the file building process for use in UNEST. Most of the above information is repeated in CAFILE if the user requests assistance in building forecasting files.

After the model has been identified and input into UNEST with the required parameters, UNEST iteratively determines the efficient parameter estimates (see Tables E.12 and E.14). Once the output has been obtained, the model can be diagnostically checked against the data used to construct the model to determine if it is appropriate. Two possible diagnostic checks built into the program are: (1) overfitting and (2) analysis of the residuals.

Overfitting involves adding an additional parameter to an estimated model and testing the hypothesis that the additional parameter equals zero. Such a test could be made using the fact that the ratio $(\hat{B}-B)/SE(\hat{B})$, where $SE(\hat{B})$ is the standard error of $\hat{B}$, is approximately normally

TABLE E.12

SUMMARY OF MODEL FOR LINEARLY INCREASING SERIES

SUMMARY OF MODEL 1

DATA - Z = FILINPOI DISTRIBUTION — 120 OBSERVATIONS

DIFFERENCING ON Z - NONE

PARAMETER NUMBER	PARAMETER TYPE	PARAMETER ORDER	ESTIMATED VALUE	95 PER CENT LOWER LIMIT	95 PER CENT UPPER LIMIT
1	AUTOREGRESSIVE 1	1	0.99998E 00	0.97045E 00	0.10295E 01
2	MEAN	1	0.73170E 03	0.57942E 07	0.57927E 07
3	TREND CONSTANT	1	0.80628E-01	0.12947E 03	0.12963E 03
4	MOVING AVERAGE 1	1	0.94503E 00	0.85559E 00	0.10345E 01

OTHER INFORMATION AND RESULTS

RESIDUAL SUM OF SQUARES	0.16720E 04	115 D.F.	RESIDUAL MEAN SQUARE	0.14539E 02
NUMBER OF RESIDUALS	119		RESIDUAL STANDARD ERROR	0.38130E 01

TABLE E.13

SUMMARY OF MODEL OF THE LINEARLY DECREASING SERIES

SUMMARY OF MODEL 1

DATA - Z = FDLINPOI DISTRIBUTION 120 OBSERVATIONS

DIFFERENCING ON Z - NONE

PARAMETER NUMBER	PARAMETER TYPE	PARAMETER ORDER	ESTIMATED VALUE	95 PER CENT LOWER LIMIT	95 PER CENT UPPER LIMIT
1	AUTOREGRESSIVE 1	1	0.99246E 00	0.12153E 20	0.12153E 20
2	MEAN	1	0.86145E 02	0.12153E 20	0.12153E 20
3	TREND CONSTANT	1	0.75132E 00	0.12153E 20	0.12153E 20
4	MOVING AVERAGE 1	1	0.99332E 00	0.10541E 12	0.10541E 12

OTHER INFORMATION AND RESULTS

RESIDUAL SUM OF SQUARES	0.31945E 04	115 D.F.	RESIDUAL MEAN SQUARE	0.27778E 02
NUMBER OF RESIDUALS	119		RESIDUAL STANDARD ERROR	0.52705E 01

TABLE E.14

SUMMARY OF MODEL FOR THE ALTERNATING LINEAR SERIES

SUMMARY OF MODEL 1

DATA - Z = FRNDLIN; DISTRIBUTION 120 OBSERVATIONS

DIFFERENCING ON Z - NONE

PARAMETER NUMBER	PARAMETER TYPE	PARAMETER ORDER	ESTIMATED VALUE	95 PER CENT LOWER LIMIT	UPPER LIMIT
1	AUTOREGRESSIVE 1	1	0.99999E 00	0.97580E 00	0.10242E 01
2	MEAN	1	0.20092E 04	0.11016E 07	0.10978E 07
3	TREND CONSTANT	1	0.13945E 00	0.42747E 02	0.43026E 02
4	MOVING AVERAGE 1	1	0.94884E 00	0.80998E 00	0.10877E 01

OTHER INFORMATION AND RESULTS

RESIDUAL SUM OF SQUARES	0.30316E 04	115 D.F.	RESIDUAL MEAN SQUARE	0.26362E 02
NUMBER OF RESIDUALS	119		RESIDUAL STANDARD ERROR	0.51344E 01

distributed, and $\hat{B}$ is some parameter estimate being tested. The hull hypothesis that $\hat{B}=0$ can be tested by comparing the above ratio with the critical value of the t distribution at the appropriate level of confidence. In the UNEST program, the confidence levels are established by the user in the parameter ICI. To test for overfitting, a simple check can be accomplished by using the summary of the model found in the output of UNEST (see Tables E.12 to E.14). The confidence intervals about the estimated value of the parameter cannot contain zero. If they do, the model is overfitted and the redundant parameters should be removed.

Analysis of the residuals $\hat{a}_t$ is a result of the idea that if the observed series is stationary, there will be no autocorrelation among the residuals. For example, a large value r_1 in the residual autocorrelation function may indicate that an addition of another moving average parameter might be justified in the model (see Tables E.15 to E.20 and Figures E.26 to E.34).

To test whether a series of autocorrelations is derived from a set of random observations (white noise), the Q statistic may be used where:

$$Q = T\sum_{j=1}^{k} r_b^2 .$$

TABLE E.15

FORECASTS FOR THE LINEARLY INCREASING SERIES

MODEL 1 FORECASTS AT BASE PERIOD 24 WITH 95 PER CENT CONFIDENCE LIMITS

PERIODS AHEAD	LO. CONF. LIMIT	FORECAST	UP. CONF. LIMIT	ACTUAL, IF KNOWN
1	0.5257731E 01	0.1271661E 02	0.2017548E 02	0.1600003E 02
2	0.5312327E 01	0.1278246E 02	0.2025258E 02	0.9000030E 01
3	0.5366941E 01	0.1284830E 02	0.2032967E 02	0.1500003E 02
4	0.5421572E 01	0.1291415E 02	0.2040674E 02	0.7000031E 01
5	0.5476220E 01	0.1298000E 02	0.2048379E 02	0.1500003E 02
6	0.5530885E 01	0.1304585E 02	0.2056082E 02	0.5000031E 01
7	0.5585568E 01	0.1311170E 02	0.2063784E 02	0.1300003E 02
8	0.5640260E 01	0.1317754E 02	0.2071483E 02	0.1300003E 02
9	0.5694969E 01	0.1324339E 02	0.2079180E 02	0.1300003E 02
10	0.5749694E 01	0.1330923E 02	0.2086876E 02	0.1200003E 02
11	0.5804436E 01	0.1337507E 02	0.2094570E 02	0.1300003E 02
12	0.5859196E 01	0.1344091E 02	0.2102263E 02	0.7000031E 01
13	0.5913971E 01	0.1350675E 02	0.2109953E 02	0.1800006E 02
14	0.5968756E 01	0.1357259E 02	0.2117642E 02	0.1000003E 02
15	0.6023557E 01	0.1363842E 02	0.2125328E 02	0.1200003E 02
16	0.6078374E 01	0.1370425E 02	0.2133013E 02	0.2000006E 02
17	0.613324[illegible]E 01	0.1377009E 02	0.2140697E 02	0.7000031E 01
18	0.6188058E 01	0.1383592E 02	0.2148379E 02	0.6000031E 01
19	0.6242924E 01	0.1390176E 02	0.2156059E 02	0.1500003E 02
20	0.6297799E 01	0.1396758E 02	0.2163737E 02	0.2100006E 02
21	0.6352690E 01	0.1403341E 02	0.2171413E 02	0.1600003E 02
22	0.6407596E 01	0.1409924E 02	0.2179087E 02	0.1400003E 02
23	0.6462519E 01	0.1416506E 02	0.2186760E 02	0.1200003E 02
24	0.6517458E 01	0.1423089E 02	0.2194432E 02	0.1600003E 02
25	0.6572412E 01	0.1429671E 02	0.2202102E 02	0.1100003E 02
26	0.6627374E 01	0.1436253E 02	0.2209769E 02	0.1400003E 02
27	0.6682352E 01	0.1442835E 02	0.2217435E 02	0.2000006E 02
28	0.6737346E 01	0.1449417E 02	0.2225100E 02	0.1600003E 02
29	0.6792355E 01	0.1455999E 02	0.2232763E 02	0.1600003E 02
30	0.6847379E 01	0.1462581E 02	0.2240424E 02	0.1700006E 02
31	0.6902412E 01	0.1469162E 02	0.2248083E 02	0.1400003E 02
32	0.6957459E 01	0.1475743E 02	0.2255740E 02	0.1900006E 02
33	0.7012522E 01	0.1482324E 02	0.2263396E 02	0.1600003E 02
34	0.7067600E 01	0.1488905E 02	0.2271051E 02	0.1500003E 02
35	0.7122693E 01	0.1495486E 02	0.2278704E 02	0.1200003E 02

TABLE E.15--Continued

44	0.7619127E 01	0.1554709E 02	0.2347505E 02	0.9000030E 01
45	0.7674353E 01	0.1561288E 02	0.2355142E 02	0.1500003E 02
46	0.7729593E 01	0.1567060E 02	0.2362777E 02	0.1800006E 02
47	0.7784848E 01	0.1574448E 02	0.2370410E 02	0.1800006E 02
48	0.7840117E 01	0.1581027E 02	0.2378043E 02	0.2000006E 02
49	0.7895393E 01	0.1587606E 02	0.2385673E 02	0.1800006E 02
50	0.7950683E 01	0.1594185E 02	0.2393301E 02	0.1900006E 02
51	0.8005988E 01	0.1600764E 02	0.2400929E 02	0.1600003E 02
52	0.8061306E 01	0.1607343E 02	0.2408554E 02	0.9000030E 01
53	0.8116639E 01	0.1613921E 02	0.2416179E 02	0.1400003E 02
54	0.8171985E 01	0.1620500E 02	0.2423802E 02	0.1700006E 02
55	0.8227338E 01	0.1627078E 02	0.2431423E 02	0.1900006E 02
56	0.8282705E 01	0.1633656E 02	0.2439042E 02	0.2000006E 02
57	0.8338085E 01	0.1640234E 02	0.2446660E 02	0.1900006E 02
58	0.8393480E 01	0.1646812E 02	0.2454277E 02	0.1700006E 02
59	0.8448888E 01	0.1653391E 02	0.2461892E 02	0.1800006E 02
60	0.8504310E 01	0.1659969E 02	0.2469506E 02	0.1100003E 02
61	0.8559738E 01	0.1666546E 02	0.2477118E 02	0.1400003E 02
62	0.8615180E 01	0.1673123E 02	0.2484728E 02	0.1300003E 02
63	0.8670635E 01	0.1679700E 02	0.2492337E 02	0.1300003E 02
64	0.8726103E 01	0.1686278E 02	0.2499945E 02	0.1600003E 02
65	0.8781585E 01	0.1692855E 02	0.2507552E 02	0.1700006E 02
66	0.8837073E 01	0.1699432E 02	0.2515156E 02	0.1300003E 02
67	0.8892574E 01	0.1706008E 02	0.2522759E 02	0.1900006E 02
68	0.894808[illegible]E 01	0.1712585E 02	0.2530361E 02	0.1500003E 02
69	0.9003616E 01	0.1719161E 02	0.2537961E 02	0.1800006E 02
70	0.9059157E 01	0.1725738E 02	0.2545560E 02	0.2200006E 02
71	0.9114711E 01	0.1732314E 02	0.2553158E 02	0.1800006E 02
72	0.9170270E 01	0.1738890E 02	0.2560753E 02	0.2100006E 02
73	0.9225843E 01	0.1745466E 02	0.2568347E 02	0.2000006E 02
74	0.9281428E 01	0.1752042E 02	0.2575940E 02	0.1700006E 02
75	0.9337027E 01	0.1758617E 02	0.2583532E 02	0.2600006E 02
76	0.9392638E 01	0.1765193E 02	0.2591123E 02	0.2200006E 02
77	0.9448242E 01	0.1771769E 02	0.2598712E 02	0.1200003E 02

TABLE E.15--Continued

86	0.9949359E 01	0.1830942E 02	0.2666948E 02	0.1500003E 02
87	0.1000509E 02	0.1837516E 02	0.2674523E 02	0.1300003E 02
88	0.1006084E 02	0.1844090E 02	0.2682096E 02	0.1900006E 02
89	0.1011660E 02	0.1850665E 02	0.2689669E 02	0.2200006E 02
90	0.1017236E 02	0.1857238E 02	0.2697240E 02	0.1500003E 02
91	0.1022814E 02	0.1863812E 02	0.2704809E 02	0.1700006E 02
92	0.1028393E 02	0.1870385E 02	0.2712377E 02	0.2000006E 02
93	0.1033973E 02	0.1876958E 02	0.2719944E 02	0.2500006E 02
94	0.1039554E 02	0.1883532E 02	0.2727510E 02	0.1900006E 02
95	0.1045137E 02	0.1890105E 02	0.2735074E 02	0.2000006E 02
96	0.1050720E 02	0.1896678E 02	0.2742637E 02	0.2700006E 02

TABLE E.16

FORECASTS FOR THE LINEARLY DECREASING SERIES

MODEL 1 FORECASTS AT BASE PERIOD 24 WITH 95 PER CENT CONFIDENCE LIMITS

PERIODS AHEAD	LO. CONF. LIMIT	FORECAST	UP. CONF. LIMIT	ACTUAL, IF KNOWN
1	0.2359431E 02	0.3388524E 02	0.4417618E 02	0.3600002E 02
2	0.2344015E 02	0.3373109E 02	0.4402203E 02	0.4300003E 02
3	0.2328716E 02	0.3357810E 02	0.4386904E 02	0.4000002E 02
4	0.2313533E 02	0.3342627E 02	0.4371721E 02	0.2700001E 02
5	0.2298463E 02	0.3327558E 02	0.4356653E 02	0.3400002E 02
6	0.2283508E 02	0.3312603E 02	0.4341698E 02	0.4200003E 02
7	0.2268665E 02	0.3297761E 02	0.4326856E 02	0.3400002E 02
8	0.2253935E 02	0.3283030E 02	0.4312126E 02	0.3200002E 02
9	0.2239315E 02	0.3268411E 02	0.4297507E 02	0.2300001E 02
10	0.2224806E 02	0.3253902E 02	0.4282999E 02	0.3500002E 02
11	0.2210406E 02	0.3239503E 02	0.4268600E 02	0.2400001E 02
12	0.2196115E 02	0.3225212E 02	0.4254309E 02	0.3100001E 02
13	0.2181931E 02	0.3211029E 02	0.4240126E 02	0.3800002E 02
14	0.2167855E 02	0.3196953E 02	0.4226051E 02	0.3900002E 02
15	0.2153885E 02	0.3182983E 02	0.4212081E 02	0.3500002E 02
16	0.2140020E 02	0.3169118E 02	0.4198217E 02	0.4300003E 02
17	0.2126260E 02	0.3155358E 02	0.4184457E 02	0.2500001E 02
18	0.2112603E 02	0.3141702E 02	0.4170801E 02	0.3300002E 02
19	0.2099050E 02	0.3128149E 02	0.4157249E 02	0.2900001E 02
20	0.2085598E 02	0.3114698E 02	0.4143798E 02	0.3500002E 02
21	0.2072249E 02	0.3101349E 02	0.4130449E 02	0.2300001E 02
22	0.2059000E 02	0.3088100E 02	0.4117200E 02	0.2000001E 02
23	0.2045851E 02	0.3074951E 02	0.4104052E 02	0.3200002E 02
24	0.2032801E 02	0.3061902E 02	0.4091003E 02	0.3000001E 02
25	0.2019849E 02	0.3048951E 02	0.4078052E 02	0.2900001E 02
26	0.2006996E 02	0.3036097E 02	0.4065199E 02	0.2800001E 02
27	0.1994239E 02	0.3023341E 02	0.4052442E 02	0.3800002E 02
28	0.1981579E 02	0.3010680E 02	0.4039782E 02	0.3300002E 02
29	0.1969014E 02	0.2998116E 02	0.4027218E 02	0.3400002E 02
30	0.1956543E 02	0.2985646E 02	0.4014748E 02	0.2900001E 02
31	0.1944167E 02	0.2973270E 02	0.4002373E 02	0.2900001E 02
32	0.1931885E 02	0.2960987E 02	0.3990090E 02	0.3900002E 02
33	0.1919694E 02	0.2948798E 02	0.3977901E 02	0.2500001E 02
34	0.1907596E 02	0.2936700E 02	0.3965803E 02	0.2500001E 02
35	0.1895589E 02	0.2924693E 02	0.3953797E 02	0.3200002E 02

TABLE E.16--Continued

44	0.1791523E 02	0.2820628E 02	0.3849734E 02	0.3000001E 02
45	0.1780391E 02	0.2809497E 02	0.3838603E 02	0.3300002E 02
46	0.1769344E 02	0.2798450E 02	0.3827556E 02	0.2500001E 02
47	0.1758380E 02	0.2787486E 02	0.3816592E 02	0.2800001E 02
48	0.1747499E 02	0.2776605E 02	0.3805712E 02	0.2400001E 02
49	0.1736700E 02	0.2765806E 02	0.3794913E 02	0.2900001E 02
50	0.1725982E 02	0.2755089E 02	0.3784196E 02	0.2400001E 02
51	0.1715345E 02	0.2744452E 02	0.3773559E 02	0.2500001E 02
52	0.1704789E 02	0.2733896E 02	0.3763003E 02	0.3500002E 02
53	0.1694312E 02	0.2723419E 02	0.3752526E 02	0.2700001E 02
54	0.1683914E 02	0.2713021E 02	0.3742129E 02	0.3100001E 02
55	0.1673594E 02	0.2702702E 02	0.3731810E 02	0.2700001E 02
56	0.1663353E 02	0.2692461E 02	0.3721568E 02	0.3900002E 02
57	0.1653189E 02	0.2682296E 02	0.3711404E 02	0.3500002E 02
58	0.1643101E 02	0.2672209E 02	0.3701317E 02	0.3200002E 02
59	0.1633089E 02	0.2662198E 02	0.3691306E 02	0.2600001E 02
60	0.1623153E 02	0.2652262E 02	0.3681370E 02	0.3200002E 02
61	0.1613292E 02	0.2642401E 02	0.3671509E 02	0.2400001E 02
62	0.1603506E 02	0.2632614E 02	0.3661723E 02	0.2300001E 02
63	0.1593793E 02	0.2622902E 02	0.3652011E 02	0.1900001E 02
64	0.1584153E 02	0.2613262E 02	0.3642371E 02	0.2500001E 02
65	0.1574586E 02	0.2603696E 02	0.3632805E 02	0.2600001E 02
66	0.1565092E 02	0.2594201E 02	0.3623310E 02	0.3000001E 02
67	0.1555668E 02	0.2584778E 02	0.3613887E 02	0.2700001E 02
68	0.1546316E 02	0.2575426E 02	0.3604536E 02	0.2400001E 02
69	0.1537035E 02	0.2566145E 02	0.3595255E 02	0.3000001E 02
70	0.1527824E 02	0.2556933E 02	0.3586043E 02	0.2300001E 02
71	0.1518682E 02	0.2547792E 02	0.3576902E 02	0.1900001E 02
72	0.1509609E 02	0.2538719E 02	0.3567829E 02	0.2600001E 02
73	0.1500604E 02	0.2529714E 02	0.3558825E 02	0.2400001E 02
74	0.1491667E 02	0.2520778E 02	0.3549888E 02	0.1300000E 02
75	0.1482798E 02	0.2511909E 02	0.3541019E 02	0.2400001E 02
76	0.1473996E 02	0.2503107E 02	0.3532217E 02	0.2700001E 02
77	0.1465260E 02	0.2494371E 02	0.3523482E 02	0.2500001E 02

TABLE E.16--Continued

86	0.1389544E 02	0.2418656E 02	0.3447768E 02	0.2100001E 02
87	0.1301445E 02	0.2410557E 02	0.3439669E 02	0.3200002E 02
88	0.1373407E 02	0.2402520E 02	0.3431632E 02	0.2300001E 02
89	0.1365430E 02	0.2394543E 02	0.3423655E 02	0.2600001E 02
90	0.1357514E 02	0.2386626E 02	0.3415738E 02	0.2500001E 02
91	0.1349656E 02	0.2378769E 02	0.3407881E 02	0.2100001E 02
92	0.1341858E 02	0.2370971E 02	0.3400083E 02	0.1700001E 02
93	0.1334119E 02	0.2363232E 02	0.3392344E 02	0.2000001E 02
94	0.1326439E 02	0.2355551E 02	0.3384664E 02	0.1800001E 02
95	0.1318816E 02	0.2347929E 02	0.3377041E 02	0.2100001E 02
96	0.1311251E 02	0.2340364E 02	0.3369476E 02	0.2000001E 02

TABLE E. 17

FORECASTS FOR THE ALTERNATING LINEAR SERIES

MODEL 1 FORECASTS AT BASE PERIOD 24 WITH 95 PER CENT CONFIDENCE LIMITS

PERIODS AHEAD	LO. CONF. LIMIT	FORECAST	UP. CONF. LIMIT	ACTUAL, IF KNOWN
1	0.1457668E 02	0.2462842E 02	0.3468016E 02	0.2400002E 02
2	0.1467731E 02	0.2474219E 02	0.3480707E 02	0.2600002E 02
3	0.1477795E 02	0.2485596E 02	0.3493396E 02	0.2400002E 02
4	0.1487862E 02	0.2496973E 02	0.3506084E 02	0.2800002E 02
5	0.1497930E 02	0.2508350E 02	0.3518769E 02	0.2700002E 02
6	0.1507998E 02	0.2519725E 02	0.3531452E 02	0.2600002E 02
7	0.1518068E 02	0.2531100E 02	0.3544133E 02	0.3100002E 02
8	0.1528140E 02	0.2542476E 02	0.3556812E 02	0.2300002E 02
9	0.1538213E 02	0.2553851E 02	0.3569489E 02	0.2600002E 02
10	0.1548289E 02	0.2565227E 02	0.3582165E 02	0.2200002E 02
11	0.1558365E 02	0.2576602E 02	0.3594839E 02	0.2900002E 02
12	0.1568444E 02	0.2587978E 02	0.3607511E 02	0.2300002E 02
13	0.1578524E 02	0.2599353E 02	0.3620182E 02	0.2300002E 02
14	0.1588606E 02	0.2610728E 02	0.3632851E 02	0.1900002E 02
15	0.1598690E 02	0.2622104E 02	0.3645518E 02	0.2400002E 02
16	0.1608775E 02	0.2633479E 02	0.3658184E 02	0.2900002E 02
17	0.1618860E 02	0.2644853E 02	0.3670846E 02	0.3100002E 02
18	0.1628947E 02	0.2656227E 02	0.3683507E 02	0.2900002E 02
19	0.1639036E 02	0.2667601E 02	0.3696166E 02	0.2500002E 02
20	0.1649126E 02	0.2678975E 02	0.3708824E 02	0.2300002E 02
21	0.1659218E 02	0.2690349E 02	0.3721480E 02	0.2800002E 02
22	0.1669312E 02	0.2701723E 02	0.3734134E 02	0.3200002E 02
23	0.1679407E 02	0.2713097E 02	0.3746786E 02	0.2800002E 02
24	0.1689504E 02	0.2724471E 02	0.3759437E 02	0.2700002E 02
25	0.1699602E 02	0.2735844E 02	0.3772087E 02	0.2000002E 02
26	0.1709702E 02	0.2747218E 02	0.3784735E 02	0.2400002E 02
27	0.1719804E 02	0.2758592E 02	0.3797381E 02	0.2300002E 02
28	0.1729905E 02	0.2769965E 02	0.3810024E 02	0.1800000E 02
29	0.1740008E 02	0.2781337E 02	0.3822666E 02	0.2800002E 02
30	0.1750113E 02	0.2792709E 02	0.3835306E 02	0.2400002E 02
31	0.1760220E 02	0.2804082E 02	0.3847944E 02	0.3100002E 02
32	0.1770328E 02	0.2815454E 02	0.3860581E 02	0.3000002E 02
33	0.1780437E 02	0.2826826E 02	0.3873216E 02	0.3000002E 02
34	0.1790548E 02	0.2838199E 02	0.3885850E 02	0.2700002E 02
35	0.1800661E 02	0.2849571E 02	0.3898482E 02	0.3600002E 02

TABLE E.17--Continued

44	0.1891732E 02	0.2951912E 02	0.4012092E 02	0.2900002E 02
45	0.1901858E 02	0.2963283E 02	0.4024707E 02	0.2700002E 02
46	0.1911986E 02	0.2974654E 02	0.4037321E 02	0.3300002E 02
47	0.1922115E 02	0.2986024E 02	0.4049934E 02	0.2500002E 02
48	0.1932246E 02	0.2997395E 02	0.4062545E 02	0.3100002E 02
49	0.1942377E 02	0.3008765E 02	0.4075153E 02	0.3100002E 02
50	0.1952509E 02	0.3020134E 02	0.4087759E 02	0.2800002E 02
51	0.1962642E 02	0.3031503E 02	0.4100364E 02	0.2700002E 02
52	0.1972777E 02	0.3042873E 02	0.4112968E 02	0.3700002E 02
53	0.1982914E 02	0.3054242E 02	0.4125570E 02	0.2600002E 02
54	0.1993052E 02	0.3065611E 02	0.4138171E 02	0.2600002E 02
55	0.2003191E 02	0.3076981E 02	0.4150770E 02	0.3200002E 02
56	0.2013332E 02	0.3088350E 02	0.4163368E 02	0.2700002E 02
57	0.2023474E 02	0.3099719E 02	0.4175964E 02	0.2800002E 02
58	0.2033618E 02	0.3111089E 02	0.4188559E 02	0.3000002E 02
59	0.2043763E 02	0.3122458E 02	0.4201153E 02	0.2900002E 02
60	0.2053908E 02	0.3133826E 02	0.4213743E 02	0.3400002E 02
61	0.2064055E 02	0.3145193E 02	0.4226332E 02	0.3300002E 02
62	0.2074202E 02	0.3156561E 02	0.4238920E 02	0.3000002E 02
63	0.2084352E 02	0.3167929E 02	0.4251507E 02	0.2000002E 02
64	0.2094502E 02	0.3179297E 02	0.4264092E 02	0.3700002E 02
65	0.2104654E 02	0.3190665E 02	0.4276675E 02	0.2900002E 02
66	0.2114808E 02	0.3202032E 02	0.4289257E 02	0.3000002E 02
67	0.2124962E 02	0.3213400E 02	0.4301838E 02	0.3300002E 02
68	0.2135119E 02	0.3224768E 02	0.4314417E 02	0.2800002E 02
69	0.2145276E 02	0.3236136E 02	0.4326996E 02	0.3000002E 02
70	0.2155434E 02	0.3247502E 02	0.4339571E 02	0.3000002E 02
71	0.2165592E 02	0.3258868E 02	0.4352144E 02	0.3600002E 02
72	0.2175753E 02	0.3270235E 02	0.4364717E 02	0.3400002E 02
73	0.2185914E 02	0.3281601E 02	0.4377288E 02	0.3200002E 02
74	0.2196077E 02	0.3292967E 02	0.4389858E 02	0.5000018E 02
75	0.2206241E 02	0.3304333E 02	0.4402426E 02	0.4600018E 02
76	0.2216407E 02	0.3315700E 02	0.4414993E 02	0.3500002E 02
77	0.2226573E 02	0.3327066E 02	0.4427559E 02	0.3500002E 02
78	0.2236742E 02	0.3338432E 02	0.4440123E 02	0.3600002E 02
79	0.2246911E 02	0.3349799E 02	0.4452686E 02	0.3200002E 02
80	0.2257082E 02	0.3361165E 02	0.4465248E 02	0.2600002E 02
81	0.2267253E 02	0.3372530E 02	0.4477806E 02	0.3300002E 02
82	0.2277425E 02	0.3383894E 02	0.4490364E 02	0.3900018E 02
83	0.2287598E 02	0.3395259E 02	0.4502920E 02	0.3300002E 02
84	0.2297773E 02	0.3406624E 02	0.4515475E 02	0.2900002E 02

TABLE E.17--Continued

[illegible]	[illegible]			
85	0.2307949E 02	0.3417989E 02	0.4528029E 02	0.3600002E 02
86	0.2318126E 02	0.3429353E 02	0.4540581E 02	0.3200002E 02
87	0.2328304E 02	0.3440718E 02	0.4553132E 02	0.3500002E 02
88	0.2338484E 02	0.3452083E 02	0.4565681E 02	0.1600000E 02
89	0.2348665E 02	0.3463448E 02	0.4578230E 02	0.3300002E 02
90	0.2358848E 02	0.3474812E 02	0.4590777E 02	0.4200018E 02
91	0.2369030E 02	0.3486176E 02	0.4603321E 02	0.4800018E 02
92	0.2379213E 02	0.3497539E 02	0.4615864E 02	0.3700002E 02
93	0.2389398E 02	0.3508902E 02	0.4628406E 02	0.4200018E 02
94	0.2399584E 02	0.3520265E 02	0.4640946E 02	0.3600002E 02
95	0.2409771E 02	0.3531628E 02	0.4653486E 02	0.3300002E 02
96	0.2419960E 02	0.3542992E 02	0.4666024E 02	0.3900018E 02

TABLE E.18

AUTOCORRELATION FUNCTION OF THE RESIDUALS FOR A LINEARLY INCREASING SERIES

ORIGINAL SERIES
MEAN OF THE SERIES =0.23426E 00
ST. DEV. OF SERIES =0.37569E 01
NUMBER OF OBSERVATIONS = 119

1- 10	0.06	0.09	0.05	-0.00	-0.14	0.02	0.03	0.06	-0.09	-0.09
ST.E.	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.10	0.10
11- 20	0.04	-0.10	-0.10	-0.08	-0.12	-0.07	0.10	0.02	0.12	0.02
ST.E.	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
21- 30	0.05	-0.06	0.11	0.07	0.11	0.11	0.13	-0.06	-0.05	-0.04
ST.E.	0.10	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.11	0.11
31- 40	-0.04	-0.06	-0.08	0.06	-0.00	-0.09	-0.01	0.00	-0.06	-0.04
ST.E.	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
41- 45	-0.04	0.00	0.10	-0.06	-0.01					
ST.E.	0.11	0.11	0.11	0.11	0.11					

MEAN DIVIDED BY ST. ERROR = 0.68022E 00

TO TEST WHETHER THIS SERIES IS WHITE NOISE, THE VALUE 0.26090E 02
SHOULD BE COMPARED WITH A CHI-SQUARE VARIABLE WITH 32 DEGREES OF FREEDOM

TABLE E.19

AUTOCORRELATION FUNCTION OF THE RESIDUALS FOR A LINEARLY DECREASING SERIES

ORIGINAL SERIES
MEAN OF THE SERIES =0.45089E 00
ST. DEV. OF SERIES =0.51833E 01
NUMBER OF OBSERVATIONS = 119

1- 10	0.01	0.02	0.07	0.10	-0.16	-0.23	-0.11	0.02	0.02	0.07
ST.E.	0.09	0.09	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.10
11- 20	0.12	0.13	0.15	-0.01	-0.05	-0.13	-0.02	-0.08	-0.08	0.00
ST.E.	0.10	0.10	0.10	0.11	0.11	0.11	0.11	0.11	0.11	0.11
21- 30	0.04	0.18	-0.05	0.06	0.07	0.09	-0.06	-0.03	-0.03	0.00
ST.E.	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
31- 40	0.04	-0.05	-0.03	0.07	-0.10	-0.11	0.03	-0.04	0.01	-0.07
ST.E.	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.12	0.12	0.12
41- 45	0.02	0.02	-0.03	0.00	-0.06					
ST.E.	0.12	0.12	0.12	0.12	0.12					

MEAN DIVIDED BY ST. ERROR = 0.94893E 00

TO TEST WHETHER THIS SERIES IS WHITE NOISE, THE VALUE 0.34064E 02
SHOULD BE COMPARED WITH A CHI-SQUARE VARIABLE WITH 32 DEGREES OF FREEDOM

TABLE E. 20

AUTOCORRELATION FUNCTION OF THE RESIDUALS OF AN ALTERNATING LINEAR SERIES

ORIGINAL SERIES
MEAN OF THE SERIES =0.24231E 00
ST. DEV. OF SERIES =0.50628E 01
NUMBER OF OBSERVATIONS = 119

1- 10	0.17	0.00	-0.06	0.06	-0.12	-0.08	-0.04	-0.10	-0.09	-0.08
ST.E.	0.09	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.10
11- 20	-0.04	-0.09	0.01	-0.00	0.16	0.16	0.15	0.05	0.03	0.01
ST.E.	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.11	0.11
21- 30	-0.06	0.05	-0.00	-0.16	0.02	0.07	-0.04	-0.08	0.09	0.08
ST.E.	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
31- 40	0.02	-0.01	-0.02	-0.09	0.12	-0.03	-0.05	-0.08	0.03	-0.07
ST.E.	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
41- 45	0.06	0.01	0.07	0.04	0.03					
ST.E.	0.11	0.11	0.11	0.11	0.11					

MEAN DIVIDED BY ST. ERROR = 0.52210E 00

TO TEST WHETHER THIS SERIES IS WHITE NOISE, THE VALUE 0.32162E 02
SHOULD BE COMPARED WITH A CHI-SQUARE VARIABLE WITH 36 DEGREES OF FREEDOM

TABLE E.21

PARTIAL AUTOCORRELATION FUNCTION OF RESIDUALS FOR A LINEARLY INCREASING SERIES

ORIGINAL SERIES
MEAN OF THE SERIES =0.23426E 00
ST. DEV. OF SERIES =0.37569F 01
NUMBER OF OBSERVATIONS = 119

1- 10	0.06	0.09	0.04	-0.02	-0.15	0.03	0.06	0.07	-0.11	-0.12
11- 20	0.07	-0.06	-0.08	-0.10	-0.12	-0.01	0.14	0.01	0.06	-0.03
21- 30	0.05	-0.05	0.13	0.06	0.04	0.10	0.09	-0.09	-0.06	-0.02
31- 40	0.01	0.00	-0.06	0.05	0.05	-0.05	-0.01	-0.01	0.05	-0.01
41- 45	-0.04	-0.05	0.09	-0.12	-0.09					

TABLE E. 22

PARTIAL AUTOCORRELATION FUNCTION OF RESIDUALS FOR A LINEARLY DECREASING SERIES

ORIGINAL SERIES
MEAN OF THE SERIES =0.45089E 00
ST. DEV. OF SERIES =0.51833E 01
NUMBER OF OBSERVATIONS = 119

1- 10	0.01	0.02	0.07	0.10	-0.17	-0.24	-0.13	0.04	0.12	0.14
11- 20	0.08	0.02	0.07	-0.03	-0.05	-0.12	0.03	0.04	0.02	0.02
21- 30	-0.06	0.10	-0.10	0.01	0.03	0.11	0.06	0.03	-0.04	-0.04
31- 40	0.09	-0.02	-0.04	0.01	-0.18	-0.12	0.03	-0.02	0.03	-0.04
41- 45	-0.05	-0.03	-0.05	-0.01	-0.02					

TABLE E.23

PARTIAL AUTOCORRELATION FUNCTION OF RESIDUALS FOR AN ALTERNATING LINEAR SERIES

ORIGINAL SERIES
MEAN OF THE SERIES =0.24231E 00
ST. DEV. OF SERIES =0.50628E 01
NUMBER OF OBSERVATIONS = 1[illegible]9

1- 10	0.17	-0.03	-0.06	0.09	-0.15	-0.03	-0.01	-0.13	-0.04	-0.08
11- 20	-0.04	-0.08	0.00	-0.03	0.14	0.10	0.08	0.03	0.00	0.02
21- 30	-0.05	0.10	0.02	-0.15	0.17	0.06	-0.03	0.03	0.10	0.03
31- 40	0.02	-0.06	-0.08	-0.09	0.18	-0.12	-0.03	-0.04	0.02	-0.07
41- 45	0.12	-0.10	0.08	0.03	-0.08					

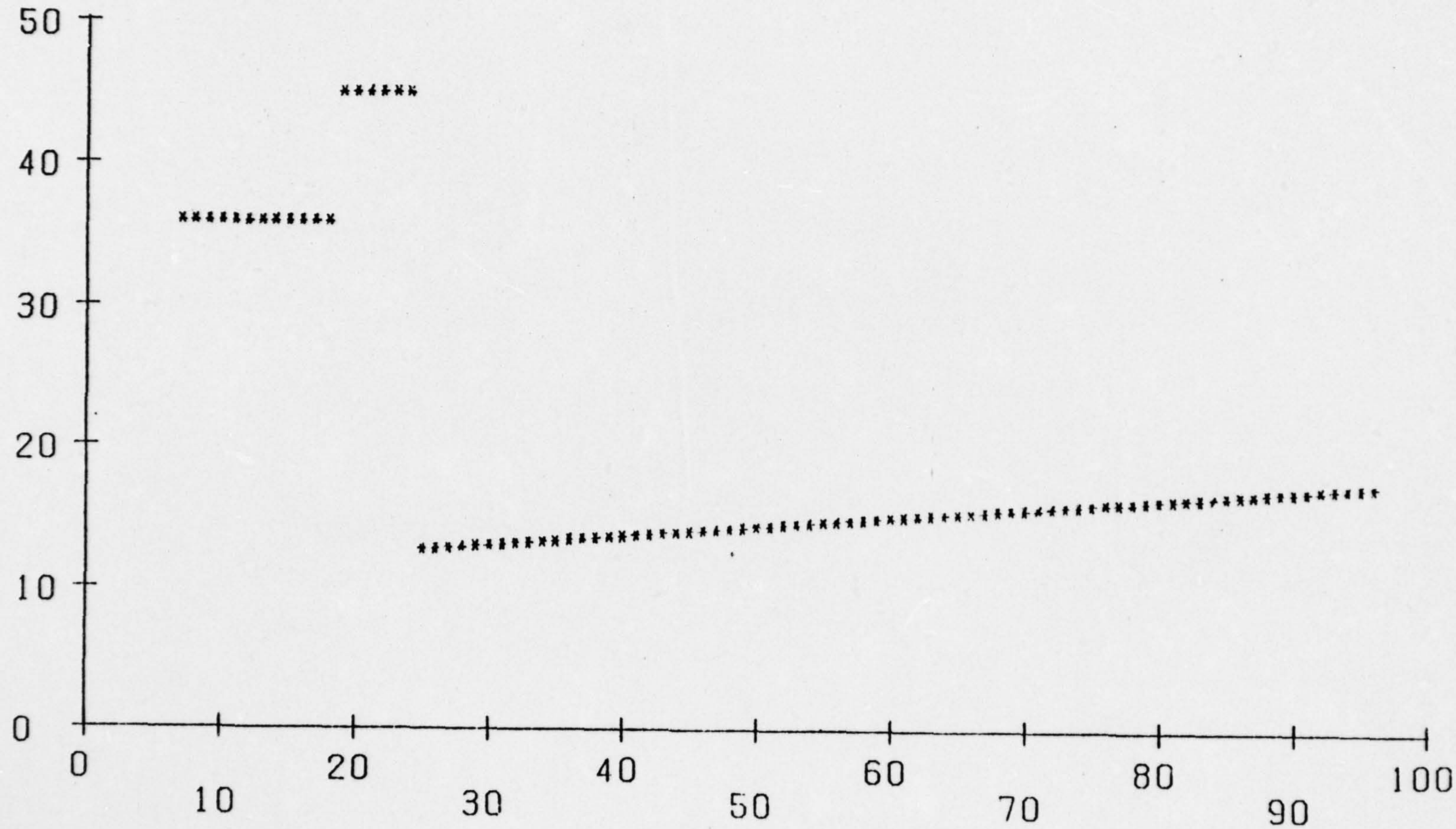


Fig. E.26. Plot of Forecasts for Linearly Increasing Poisson Process

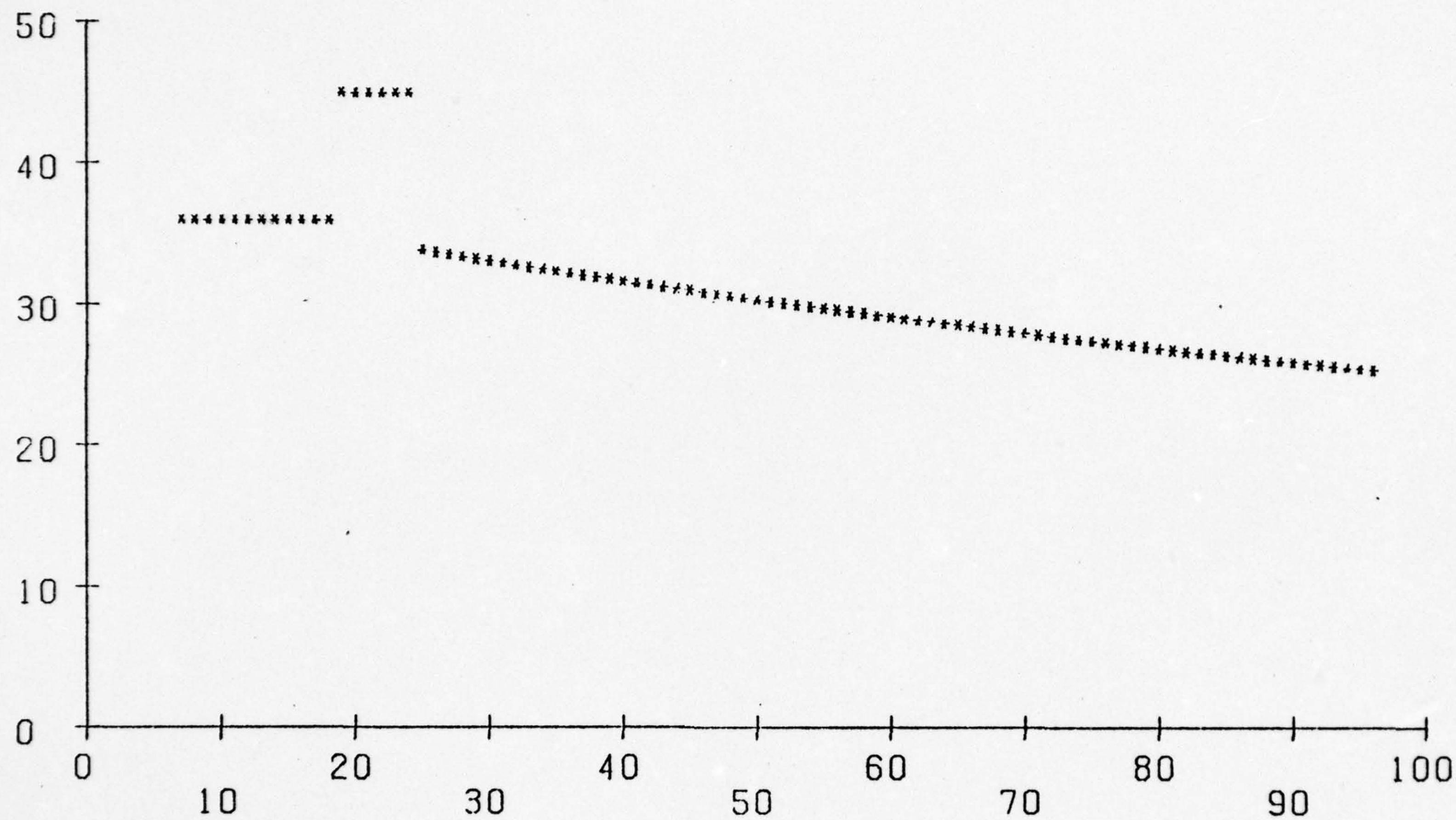


Fig. E.27. Plot of Forecasts for Linearly Decreasing Poisson Process

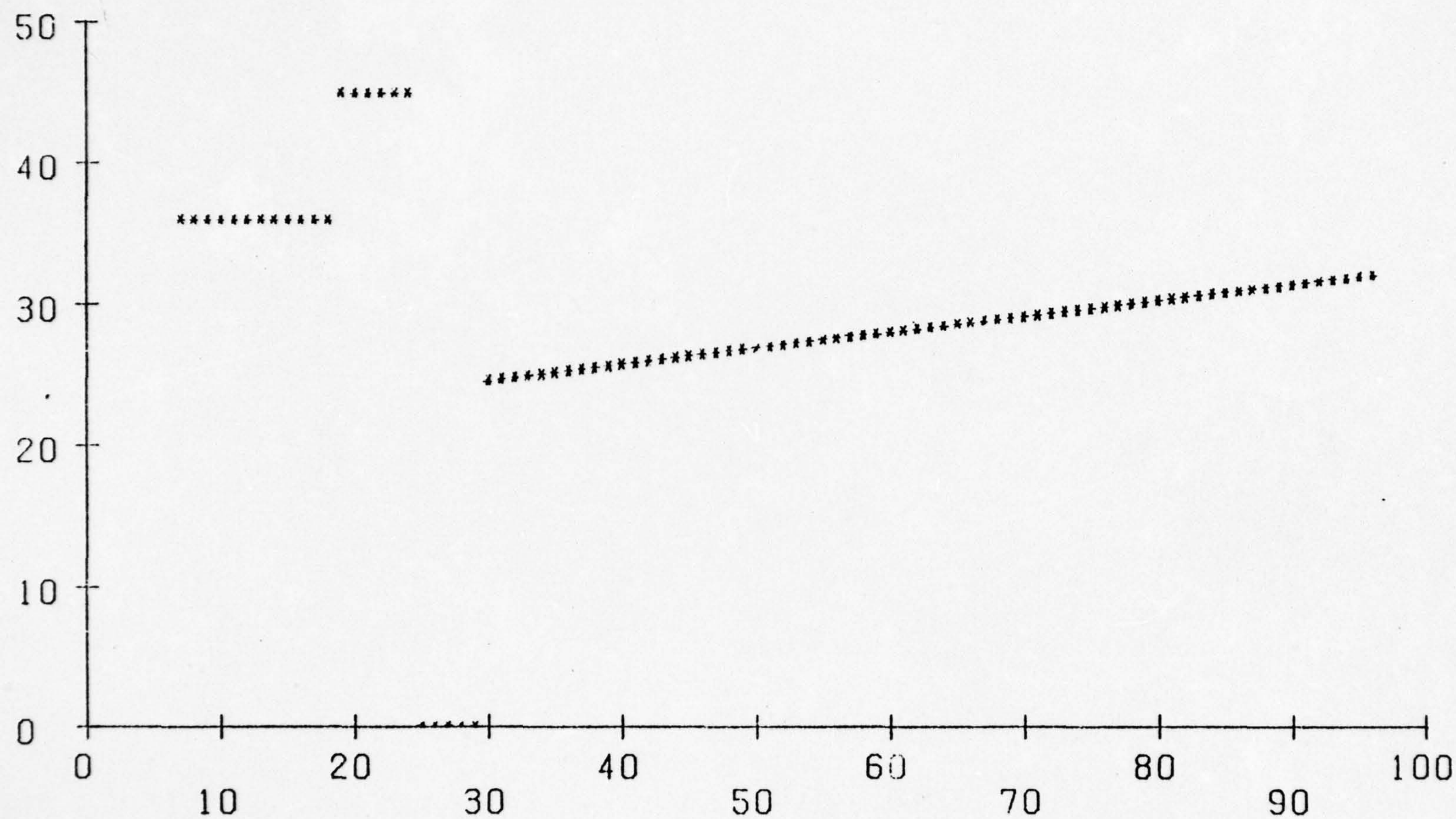


Fig. E.28. Plot of Forecasts for Alternating Linear Poisson Process

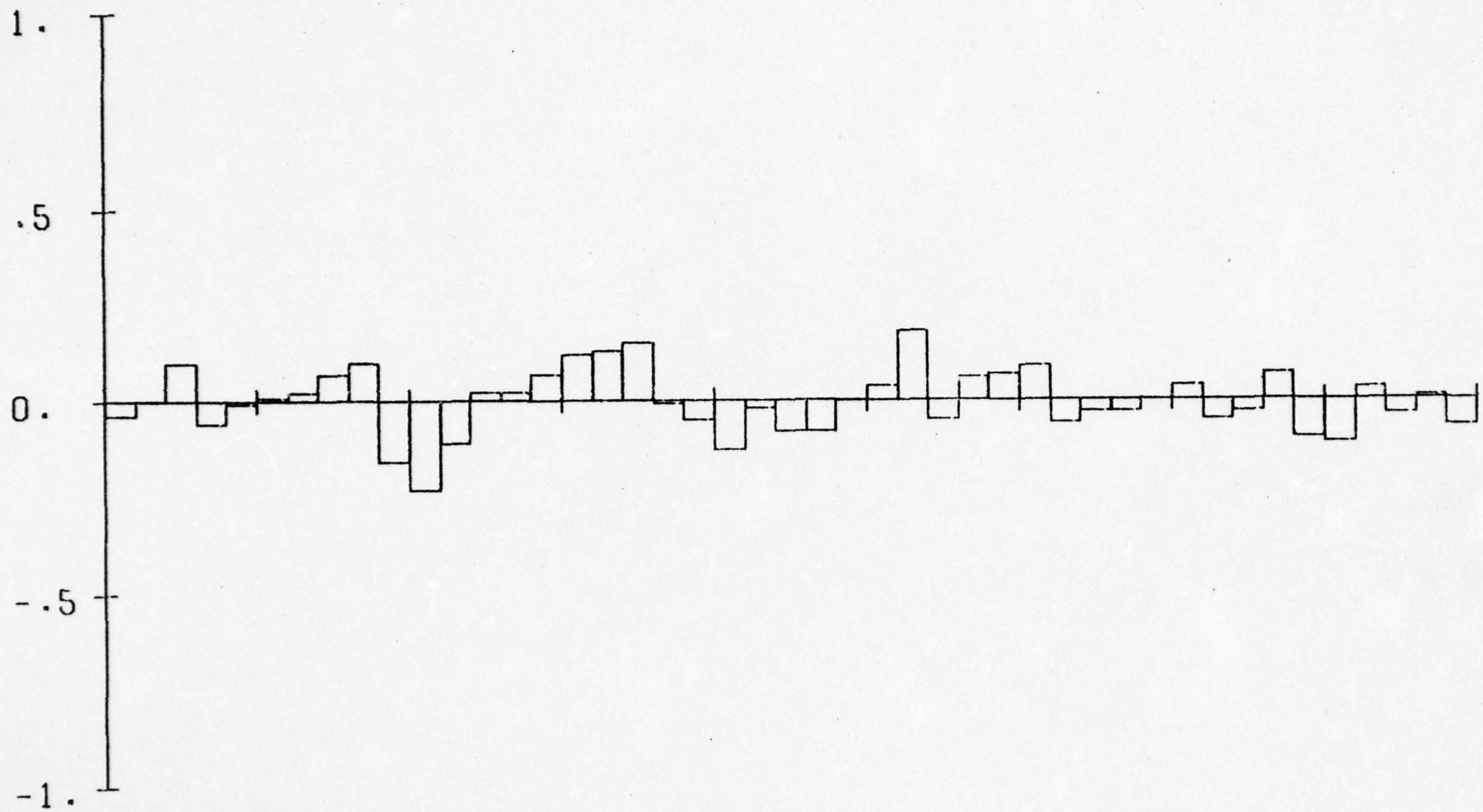


Fig. E.29. ACF of Residuals for Linearly Increasing Series

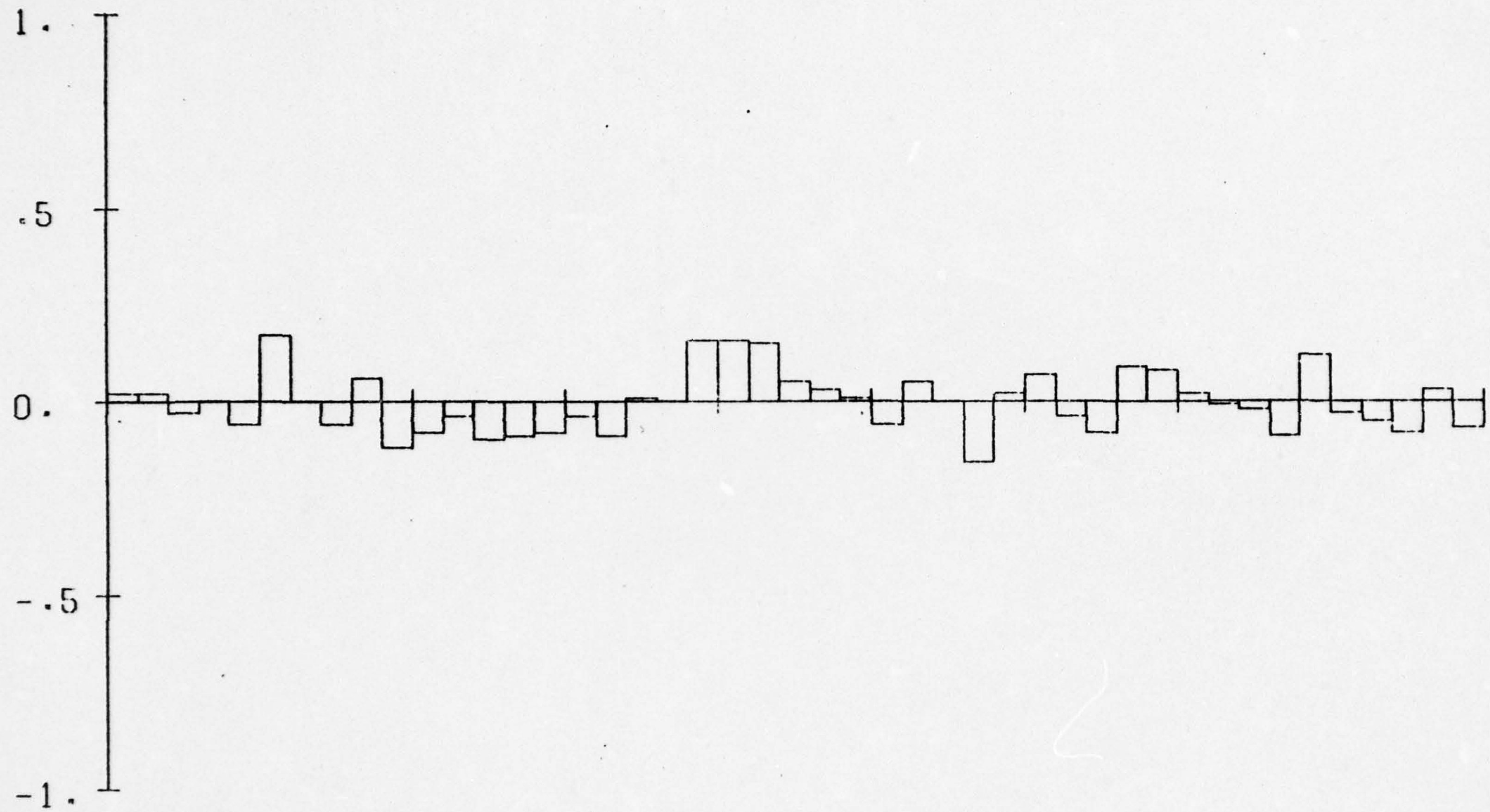


Fig. E.30. ACF of Residuals for Linearly Decreasing Series

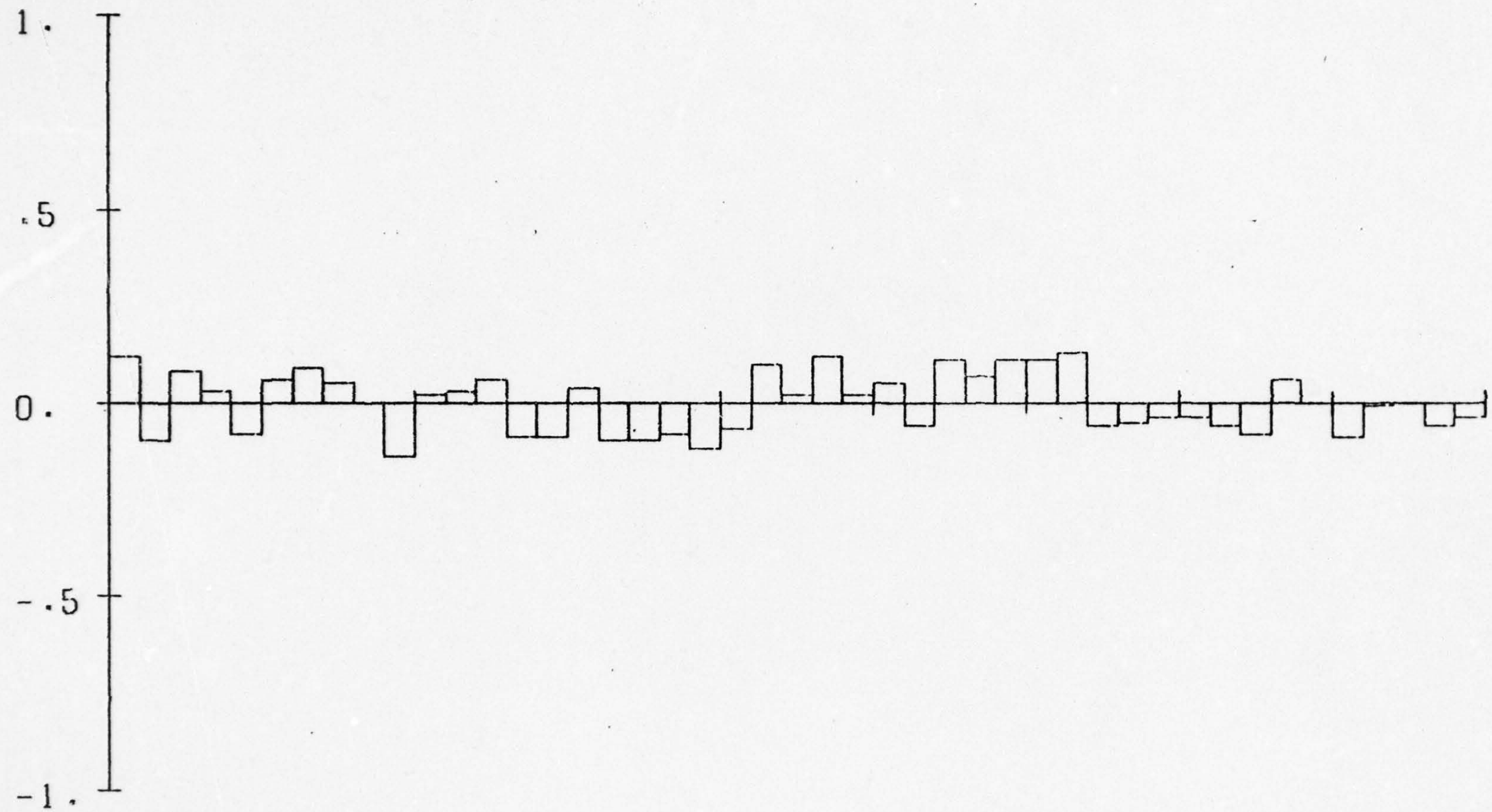


Fig. E.31. ACF of Residuals for Alternating Series

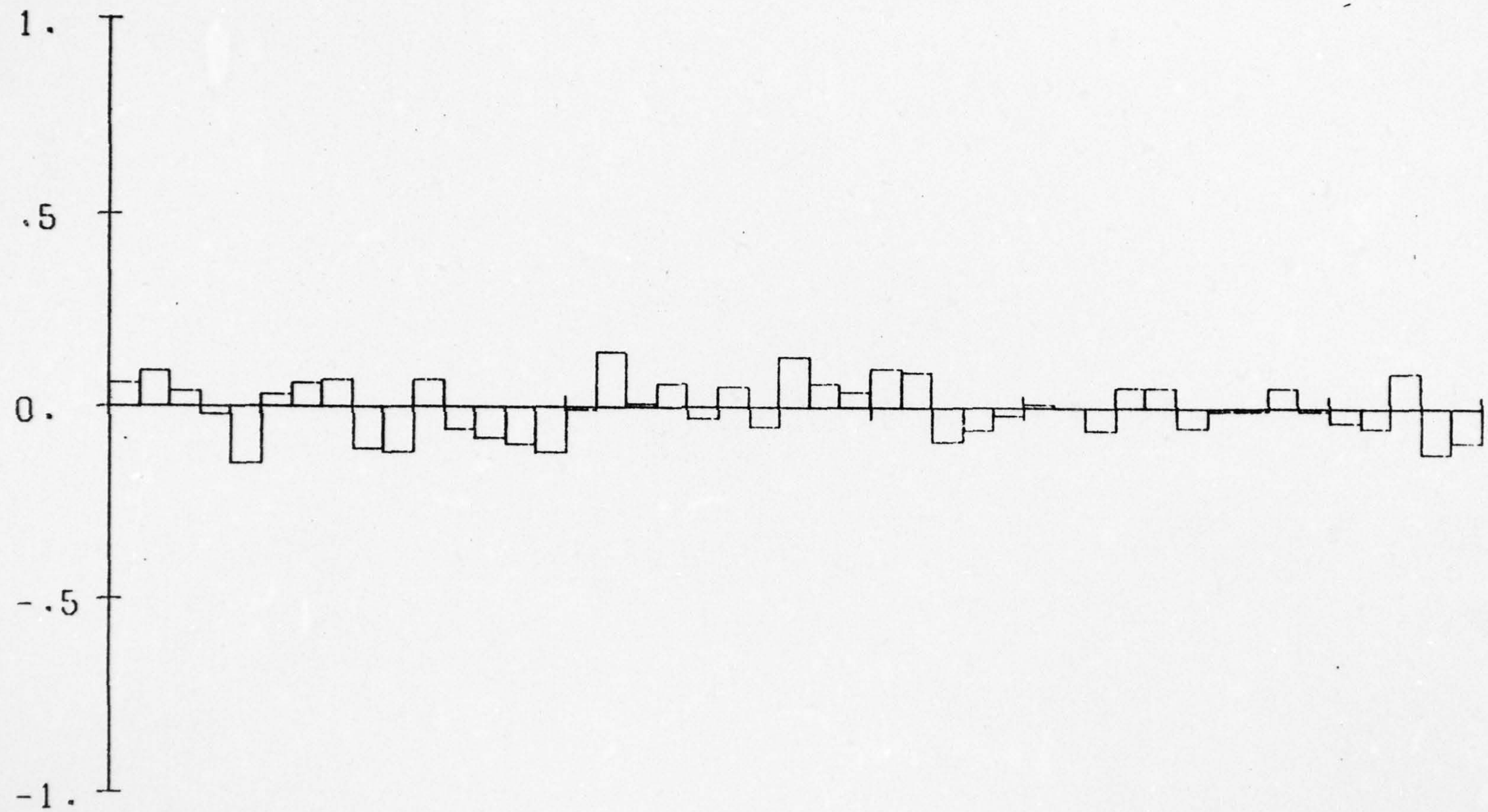


Fig. E.32. PACF of Residuals for Linearly Increasing Poisson Process

AD-A032 364 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OHIO SCHO--ETC F/G 15/5
A COMPARATIVE ANALYSIS OF THE D041 SYSTEM AND TIME SERIES ANALY--ETC(U)
SEP 76 B R CHRISTENSEN, G J SCHROEDER

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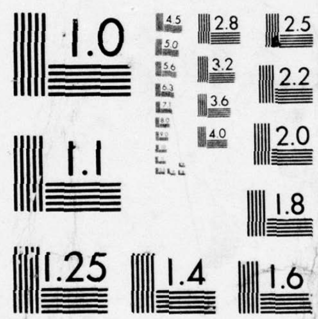

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

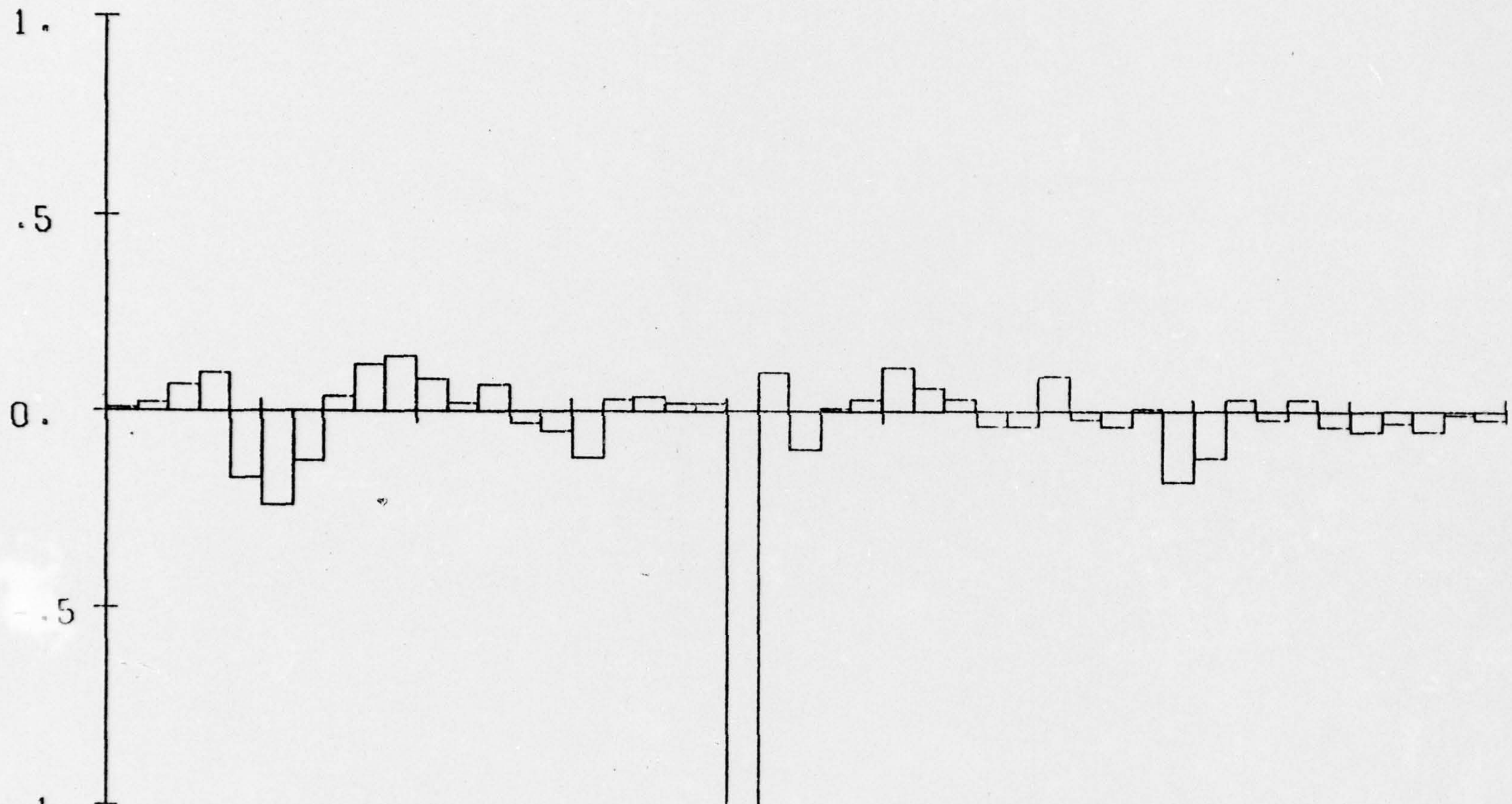

Fig. E.33. PACF of Residuals for Linearly Decreasing Poisson Process

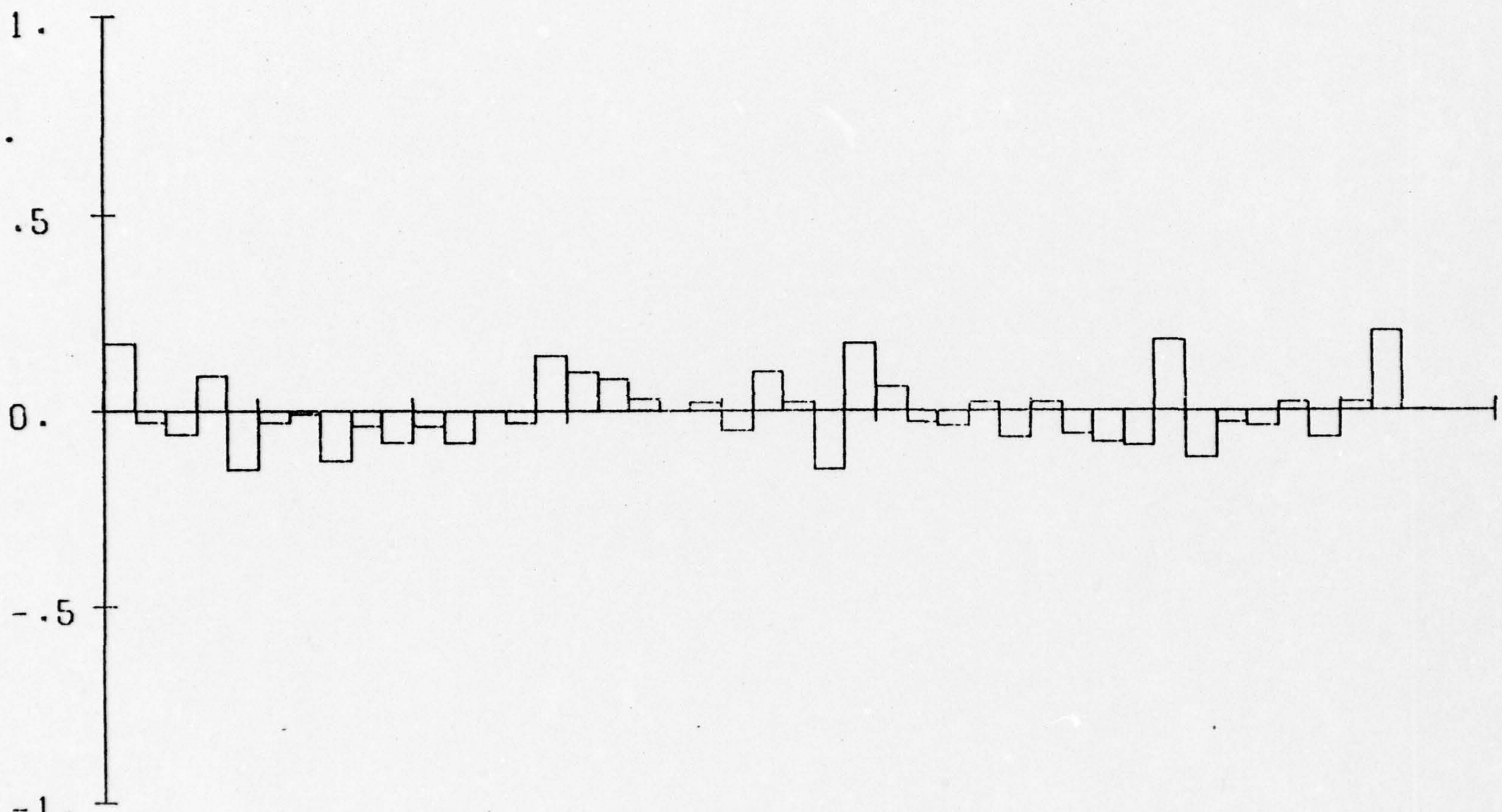

Fig. E.34. **PACF of Residuals for Alternating Linear Poisson Process**

The computed value of Q may be compared with the Chi-square value having k-p-q degrees of freedom to determine if a set of sample autocorrelations is significantly large. Using UNEST, the Chi-square value calculated on the autocorrelation function of the residuals is compared with the Chi-square critical value obtained from Chi-square tables at an appropriate confidence level (refer to Tables E.18 to E.20 and Figures E.29 to E.31). If the computed Chi-square value is less than the critical Chi-square value, it can be concluded that the autocorrelation among the residuals is not significant and there is no indication of a lack of fit.

Should either of the the two diagnostic checks fail, the model identification process should be repeated. The Box and Jenkins time series analysis for forecasting methods are not a hands-off technique for forecasting future observations. The user of time series analysis is required to follow the above procedures, iteratively, until the correct model is built to forecast future observations.

The time series models are useful in the model building process, i.e., building a model to fit the data. Once the model fits, forecasts should be excellent, exhibiting no bias (see Tables E.15 to E.20 and Figures E.26 to E.28). Time series analysis techniques also offer the advantage of being able to calculate confidence intervals about the forecasted observations.

APPENDIX F

RESULTS OF STATISTICAL HYPOTHESIS TEST

APPENDIX F

RESULTS OF STATISTICAL HYPOTHESIS TEST

Complete Outputs of Single Moving Average Forecasts

This appendix contains the tables of outputs of the single moving average forecasts and the time series analysis forecast. For each data pattern, the following information is given:

1. Thirty-six forecasts of each demand pattern

2. The sum of the forecast errors associated with each lead time

3. The average forecast error for each lead time (the forecast bias)

4. The standard deviation of each run of forecast errors

5. The statistic:

$$\frac{\text{Average Forecast Error}}{\text{Standard Deviation/36}} .$$

TABLE F.1

OUTPUT OF POISSON PATTERN: MEAN 10

36 Single Moving Average Forecasts					
9.9167	10.000	9.9167	10.2917	10.3333	10.4167
10.5000	10.7083	10.7083	10.5833	10.483	10.4167
10.2917	10.2083	10.1667	10.2500	10.2500	10.5833
10.4583	10.5417	10.7083	10.3750	10.2083	9.8750
10.0000	9.7500	10.2083	10.0417	9.9167	9.9583
9.9583	9.9167	9.9167	9.9583	9.9583	10.2083

TABLE F.2

STATISTICAL RESULTS OF POISSON SERIES AT DIFFERENT LEAD TIMES

Lead Time	Error Sum	Bias	Standard Deviation	Test Statistic
1	-0.0417	-0.0012	3.3402	-0.0021
3	-2.0417	-0.0567	3.2499	-0.1047
6	1.9583	0.0544	3.1362	0.1041
9	-4.0417	-0.1123	3.1430	-0.2143
12	-5.0417	-0.1400	3.2071	-0.2620
15	-2.0417	-0.0567	3.2108	-0.1060
18	-3.0417	-0.9845	2.9453	-0.1721
21	-5.0417	-0.1400	3.0597	-0.2746
24	-7.0417	-0.1956	3.0246	-0.3874
27	3.9583	0.1100	2.6237	0.2514
30	-2.0417	-0.0567	2.5081	-0.1357
33	-9.0417	-0.2512	2.5214	-0.5977
36	-5.0417	-0.1400	2.3804	-0.3530

TABLE F.3

OUTPUT OF LINEARLY INCREASING POISSON PATTERN: MEAN BEGINS AT 10 FOR THE FIRST DATA POINT AND ENDS AT 20 FOR THE LAST DATA POINT

36 Single Moving Average Forecasts

10.9167	11.0417	11.0000	11.3333	11.2917	11.6250
11.5000	11.6667	11.7917	11.9167	11.7500	11.8333
11.6250	11.9167	11.7917	12.0417	12.5417	12.2500
12.1250	12.2500	12.4167	12.5417	12.6667	12.5833
12.7083	12.5000	12.7083	12.9167	13.2917	13.3333
13.8333	13.8750	14.1250	14.2500	14.3750	14.3333

TABLE F.4

STATISTICAL RESULTS OF LINEARLY INCREASING SERIES AT DIFFERENT LEAD TIMES

Lead Time	Error Sum	Bias	Standard Deviation	Test Statistic
1	-40.333	-1.204	3.8808	-1.7322
3	-46.333	-1.7870	3.8376	-2.0177
6	-59.333	-1.6481	3.7398	-2.6447
9	-63.333	-1.7593	3.8793	-2.7210
12	-76.333	-2.1204	3.8636	-3.2929
15	-98.333	-2.715	3.5932	-4.5611
18	-98.333	-2.7315	3.5492	-4.6177
21	-112.333	-3.1204	3.1398	-5.9629
24	-124.333	-3.4537	3.0693	-6.7514
27	-121.333	-3.3704	3.3839	-5.9760
30	-155.33	-3.2037	3.3362	-5.76117
33	-112.333	-3.1204	3.2574	-5.7476
36	-127.333	-3.5370	3.2947	-6.4414

TABLE F.5

OUTPUT OF LINEARLY DECREASING POISSON: MEAN BEGINS AT 40 FOR THE FIRST DATA POINT AND ENDS AT 20 FOR THE LAST DATA POINT

36 Single Moving Average Forecasts

37.8333	37.7500	38.2917	27.9167	37.2917	37.2917
37.0417	36.7917	36.3333	35.9583	35.8333	35.5417
35.1667	35.3750	35.1667	35.1667	35.3333	34.5000
34.3750	33.8750	33.9583	33.6250	32.7500	32.5417
32.6250	32.3333	31.7083	31.6250	31.8750	31.8750
31.3333	31.1250	31.4167	31.5000	31.0833	31.4167

TABLE F.6

STATISTICAL RESULTS OF LINEARLY DECREASING SERIES AT DIFFERENT LEAD TIMES

Lead Time	Error Sum	Bias	Standard Deviation	Test Statistic
1	86.6250	2.4063	5.7637	2.5049
3	106.6250	2.9618	5.5532	3.2001
6	121.6250	3.3785	5.6996	3.5565
9	148.6250	4.1285	5.5918	4.4299
12	144.6250	4.0174	5.0051	4.8159
15	175.6250	4.8785	5.1203	5.7166
18	191.6250	5.3229	4.9096	6.5051
21	191.6250	5.3229	5.3172	6.0064
24	173.6250	4.8229	4.9029	5.9021
27	181.6250	5.0451	5.0377	6.0089
30	216.6250	6.0174	4.1752	6.9764
33	232.6250	6.4618	5.0116	4.8120
36	242.6250	6.7396	4.8120	8.4034

TABLE F.7

OUTPUT OF ALTERNATING LINEAR SERIES POISSON PATTERN: MEAN ORIGINALLY SET AT 20 AND THEN INCREASES FOR 12 DATA POINTS AND THEN DECREASES FOR 6 DATA POINTS AND THEN REPEATS INCREASING AND DECREASING IN SAME PATTERNS

36 Single Moving Average Forecasts					
22.3750	22.3750	22.7917	23.2500	23.6250	23.6667
23.6667	23.9583	24.2500	24.4583	24.1667	24.2083
24.0417	24.0000	23.7917	23.8333	24.3333	24.7083
25.0000	25.3750	25.2917	25.5000	25.9583	26.3750
25.1250	25.9583	25.8750	25.8333	25.4167	25.4583
25.3750	25.3750	25.6667	25.8333	25.0417	25.3333

TABLE F.8

STATISTICAL RESULTS OF ALTERNATING LINEAR SERIES AT DIFFERENT LEAD TIMES

Lead Time	Error Sum	Bias	Standard Deviation	Test Statistic
1	-49.7083	-1.3808	4.0107	-2.0657
3	-54.7083	-15.197	4.0929	-2.2278
6	-52.7083	-1.4641	4.4057	-1.9939
9	-52.7083	-1.4641	4.0864	-2.1497
12	-60.7083	-1.6863	3.9587	-2.5559
15	-85.7083	-2.3808	4.0947	-3.4886
18	-91.7083	-2.5475	4.3702	-3.4967
21	-99.7083	-2.7697	4.3407	-3.8284
24	-98.7083	-2.7419	3.9847	-4.1286
27	-124.7083	-3.4641	3.8577	-5.3878
30	-141.7083	-3.9363	4.2133	-5.6056
33	-147.7083	-4.1030	4.2193	-5.8347
36	-150.7083	-4.1863	3.9492	-6.3602

TABLE F.9

OUTPUT OF SINE PATTERN: AMPLITUDE IS SET AT 20 AND THE MEAN IS SET AT 50

36 Single Moving Average Forecasts					
55.9602	56.1520	57.5298	58.2046	48.4816	57.3048
57.1007	57.5828	57.5828	59.2406	58.4371	58.8171
57.1583	57.3623	56.5650	55.2872	55.2872	54.2228
55.2237	53.9459	52.6742	52.8467	53.0672	51.9180
50.2602	50.0617	59.3621	47.9869	46.3633	46.7495
46.7495	46.2742	46.2742	45.7931	45.7931	46.0066

TABLE F.10

STATISTICAL RESULTS OF THE SINE SERIES AT DIFFERENT LEAD TIMES

Lead Time	Error Sum	Bias	Standard Deviation	Test Statistic
1	106.7761	2.9660	14.8289	1.2001
3	141.8972	3.9416	14.7415	1.6043
6	172.2818	4.7856	14.4335	1.9894
9	190.4720	5.2909	14.7721	2.1490
12	192.0134	5.3337	14.7074	2.1759
15	152.2264	4.2285	15.0838	1.6820
18	199.4587	5.5405	14.6222	2.2735
21	196.7626	5.4656	14.5475	2.2543
24	171.3871	4.7608	14.5946	1.9572
27	154.5975	4.2944	15.0652	1.7103
30	127.1646	3.5323	14.3265	1.4794
33	174.5166	4.8477	14.7789	1.9681
36	179.4900	4.9858	14.9835	1.9965

TABLE F.11

OUTPUT OF LINEARLY INCREASING POISSON PATTERN TIME SERIES FORECASTING RESULTS

Lead Time	Error Sum	Bias	Standard Deviation	Test Statistic
1	12.2740	.3409	3.8075	.5373
3	11.0137	.3059	3.8077	.4821
6	5.1226	.1423	3.7529	.2275
9	8.2314	.2287	3.776	.3632
12	2.3397	.0650	3.8187	.1021
15	-12.5524	-.3487	3.6367	-.5753
18	- 5.448	-.1512	3.5169	-.2580
21	-12.3377	-.3427	3.1119	-.6608
24	-17.2310	-.4786	3.0962	-.9275
27	- 7.1248	-.1979	3.2977	-.3601
30	5.9811	.1661	3.2108	.3105
33	16.0864	.4468	3.1837	.8421
36	8.1914	.2275	3.2669	.4179

TABLE F.12

OUTPUT OF LINEARLY DECREASING POISSON TIME SERIES FORECASTING RESULTS

Lead Time	Error Sum	Bias	Standard Deviation	Test Statistic
1	-12.5007	-.5139	5.6922	-.5417
3	- 8.2154	-.2282	5.4838	-.2497
6	- 7.5143	-.2087	5.5058	-.2275
9	5.5079	.1530	5.5039	.1668
12	-12.1560	-.3317	5.1270	-.03952
15	5.4870	.1524	4.9738	.1839
18	8.4301	.2342	4.6246	.3038
21	- 4.2701	-.1187	5.0385	-.1412
24	-34.7472	-.9652	4.6795	-1.2376
27	-38.9439	-1.0818	4.7707	-1.3605
30	-15.8667	-.4407	4.7822	-.5530
33	-11.5217	-.3200	4.5831	-.4190
36	-12.9152	-.3588	4.6439	-.4635

TABLE F.13

OUTPUT OF ALTERNATING LINEAR INCREASING AND DECREASING POISSON TIME SERIES FORECASTING RESULTS

Lead Time	Error Sum	Bias	Standard Deviation	Test Statistic
1	18.2847	.5079	3.9042	.7806
3	21.4741	.5965	3.8976	.9183
6	35.7376	.9933	4.2802	1.3924
9	48.0407	1.3349	4.1542	1.9274
12	52.3237	1.4534	4.1720	2.0903
15	39.6061	1.1002	4.0730	1.7207
18	45.8881	1.2747	4.1671	1.83333
21	50.1646	1.3935	4.1798	2.0003
24	63.4456	1.7624	3.9926	2.6485
27	49.7261	1.3813	3.9523	2.0969
30	45.0060	1.2502	4.1951	1.7850
33	51.2856	1.4246	4.1212	2.0740
36	60.5650	1.6824	3.8631	2.5995

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